Cognitive Uncertainty[*]

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Abstract

This paper documents the economic relevance of measuring *cognitive uncertainty*: people's subjective uncertainty over their ex-ante utility-maximizing decision. In a series of experiments on choice under risk, the formation of beliefs and forecasts of economic variables, we show that cognitive uncertainty predicts various systematic biases in economic decisions. When people are cognitively uncertain – either endogenously or because the problem is designed to be complex – their decisions are heavily attenuated functions of objective probabilities, which gives rise to average behavior that is regressive to an intermediate option. This insight ties together a wide range of empirical regularities in behavioral economics that are typically viewed as distinct phenomena or even as reflecting preferences, including the probability weighting function in choice under risk; base rate insensitivity, conservatism and sample size effects in belief updating; and predictable overoptimism and -pessimism in forecasts of economic variables. Our results offer a blueprint for how a simple measurement of cognitive uncertainty generates novel insights about what people find complex and how they respond to it.

Keywords: Cognitive uncertainty, complexity, cognitive noise, beliefs, expectations, choice under risk

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1 Introduction

This paper studies the economic relevance of *cognitive uncertainty*: people's subjective uncertainty over which decision maximizes their expected utility. In the standard economic model, people take decisions that they know may turn out to be ex-post suboptimal, but they never exhibit doubts about their ex-ante optimality. Similarly, in a large majority of behavioral economics models, people may make systematic mistakes, but they are not nervous that they may be committing errors. Yet, both introspection and a growing body of psychological evidence discussed below suggest that people often exhibit low confidence in their decisions. However, it is not immediately obvious why the insight that people have some meta-cognitive awareness of their own decision errors should be relevant to the interests of economists in formally modeling and predicting behavior.

This paper proposes that measuring cognitive uncertainty can be productively deployed to predict systematic biases in economic behaviors and to help tie together widely-studied behavioral economics anomalies that are typically viewed as distinct phenomena. The main idea consists of two components. (i) Classical anomalies share a common origin, which is that the inherent complexity of economic decisions induces people to make noisy or heuristic decisions instead of solving a problem precisely. These simpler decision modes produce behaviors that are severely attenuated functions of objective problem parameters and are regressive to an intermediate option. (ii) Cognitive uncertainty represents an easily-measurable proxy for the unobserved noisiness or heuristic nature of people's decision modes, and can thus be used to predict and explain behavior.

We present experiments on decision-making under uncertainty: the ways people reason about probabilities in the valuation of risky lotteries, inference from data, and prediction of future events. As Figure 1 illustrates using our own experimental data, these three literatures have established striking similarities about how objective probabilities map into people's decisions. First, the left-hand panel depicts the well-known probability weighting function in choice under risk that goes back to Tversky and Kahneman (1992). It illustrates how experimental subjects implicitly treat objective probabilities in choosing between different monetary gambles. Second, the middle panel illustrates the canonical compressed relationship between participants' posterior beliefs and the Bayesian posterior in experimental belief updating tasks, which shows that people generally overestimate the probability of unlikely events and underestimate the probability of likely ones. Finally, the right-hand panel shows the compressed relationship between respondents' probabilistic estimates and "true" probabilities that has been documented in a wide range of subjective expectations

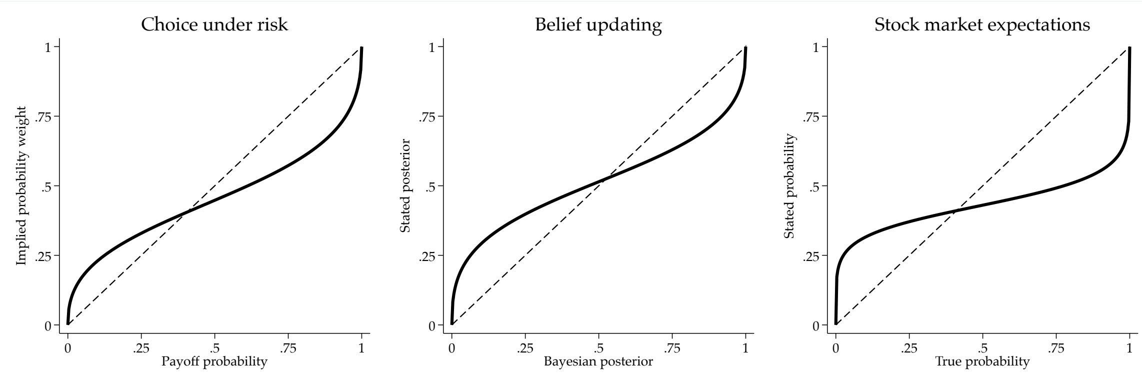

Figure 1: Decisions as functions of objective probabilities. The left-hand panel illustrates a probability weighting function in choices between monetary gambles. The middle panel illustrates the relationship between stated beliefs and Bayesian posteriors inbelief updating experiments. The right-hand panel illustrates the typical relationship between stated subjective probabilities and objective (historical) probabilities in surveys about stock returns or inflation. All functions are estimated from the data discussed in Appendix E.

surveys about, for example, stock market returns or inflation rates. The characteristic feature of these three functions is that people's decisions implicitly treat different probabilities to some degree alike, which generates a compression effect to an "intermediate" value.

One view in the literature – reflected in the existence of a large number of dedicated models of probability weighting and belief updating – is that these phenomena reflect domain-specific biases or even preferences. Another view is that they have a common origin: in response to the inherent complexity of forming beliefs and choosing between lotteries, people may deploy simpler, noisy or heuristic decision modes. For example, in recent Bayesian cognitive noise models of choice under risk, the difficulty of translating objective probabilities into decisions introduces cognitive noise, which induces the decision-maker to partially regress to (or anchor on) an intermediate cognitive default, thus producing probability weighting through a mechanism akin to the classical anchoring-and-adjustment heuristic. Similarly, systematic compression to an intermediate value can result if people choose randomly with some probability. Regardless of what exactly the underlying decision mode is, this class of models highlights that random noise often generates systematic bias. At the same time, there is little evidence that directly ties together and explains behavior across the three decision domains in Figure 1 as a function of noisy cognition and complexity.

To make progress, we measure cognitive uncertainty as a proxy for the inherent noisiness or heuristic nature of people's decisions. We conduct a series of online experiments with a total of more than 3,000 participants. We elicit entirely standard controlled decisions in each of the three domains discussed above. In addition to eliciting payoff-relevant choices and beliefs, we also measure cognitive uncertainty. For example, in lottery valuation tasks, after we elicit a participant's certainty equivalent, we ask them how certain they are (in percent) that their true valuation of the lottery actually lies within a one-dollar window around their stated valuation. Similarly, after participants state probabilistic beliefs in canonical belief updating experiments, we ask them how certain they are that the Bayesian posterior is contained in a two percentage point window around their stated belief. These questions elicit people's subjective percent chance that their decision is actually (close to) the ex-ante utility-maximizing one.

This cognitive uncertainty elicitation has five main features. (i) The measure admits a direct theoretical interpretation of awareness of noise. (ii) As documented by our three applications, the elicitation can be tweaked in minor ways to be applicable to a broad set of decision domains with very different experimental paradigms and elicitation protocols. (iii) The question is a composite measure that potentially captures people's awareness of a multitude of cognitive imperfections, such as imperfect perception, preference uncertainty, problems in integrating utils and probabilities, lack of knowledge of Bayes' rule, computational difficulties or memory imperfections. As a result, a productive interpretation of cognitive uncertainty is that it captures people's subjective difficulty or perceived complexity of a problem. (iv) The measure is very simple, quick and costless to elicit, making it easy for researchers to add such a question to their own studies. (v) Cognitive uncertainty is strongly correlated with decision variability in repetitions of the same decision problem, which is a key choice signature of (cognitive) noise.

We find large variation in cognitive uncertainty in all of our decision domains. In choice under risk, more than 80% of all decisions are associated with strictly positive cognitive uncertainty, and this number rises to more than 90% in belief updating. Participants appear relatively consistent in their degree of cognitive uncertainty, both across repeated decisions within the same domain ($r \approx 0.7$) and across different decision domains.

Measured cognitive uncertainty strongly predicts observed choices and beliefs in a way that sheds light on the empirical anomalies summarized in Figure 1. In all three decision domains, high cognitive uncertainty decisions are substantially more compressed and less responsive to variation in objective probabilities. For example, in choice under risk, high cognitive uncertainty decisions exhibit a substantially shallower slope of the probability weighting function, which implies that cognitive uncertainty is strongly correlated with the well-known fourfold pattern of risk attitudes. For decisions with cognitive uncertainty of zero, the median decision exhibits essentially no probability weighting.

In the domains of beliefs and expectations, we likewise see that high cognitive uncertainty beliefs are substantially more compressed towards 50:50. This means that cognitively uncertain people will sometimes appear more optimistic and sometimes more pessimistic than is warranted, purely depending on whether the true probability is high or low. Cognitive uncertainty is also strongly predictive of more structural belief updating biases, including base rate insensitivity and conservatism. We discuss implications of these results for interpreting heterogeneity in economic expectations surveys.

The predictive power of cognitive uncertainty for compression effects in decisions is not only driven by the extensive margin of cognitive uncertainty. Instead, the link is strictly monotonic: people in the lowest cognitive uncertainty quartile respond more to objective probabilities than people in the second quartile, who in turn respond more than those in the third quartile, and so on. This shows that the magnitude of cognitive uncertainty contains much information even away from the rational benchmark of zero, and that strictly positive cognitive uncertainty is not just driven by measurement error.

We are agnostic over whether the strong correlations between cognitive uncertainty and behaviors reflect a causal effect of the true (cognitive or decision) noise that underlies cognitive uncertainty or whether awareness of potential errors itself drives behaviors. Under either interpretation, our hypothesis is that the link between cognitive uncertainty and decisions partly reflects the complexity of identifying the utility-maximizing decision. To directly investigate this complexity interpretation, we implement different treatments that vary the complexity of the lottery valuation and belief updating tasks. In one set of experiments, we vary the computational complexity of the decision problems by displaying the relevant problem parameters (such as payout probabilities or base rates) as algebraic expressions. In other experiments, we increase problem complexity by turning lotteries or belief updating tasks into compound (multi-stage probabilistic) problems.

We always find that higher complexity leads to higher cognitive uncertainty, which lends credence to our interpretation that cognitive uncertainty partly reflects the subjectively perceived complexity of decision problems. Moreover, the compression effects summarized in Figure 1 become substantially more pronounced in the more complex treatments. For instance, contrary to the predictions of (cumulative) prospect theory, the probability weighting function exhibits substantially stronger likelihood insensitivity when the decision problems are more complex. Similarly, in contrast to models of base rate neglect or conservatism that rest on assumptions of fixed parametric biases, the magnitude of base rate insensitivity and conservatism strongly depends on the complexity of the decision problem.

To sum up, this paper documents that cognitive uncertainty can be effectively used to test hypotheses about cognitive or decision noise that are difficult to test otherwise. Our results highlight that various judgment and decision errors that are traditionally viewed as distinct share, in fact, common cognitive origins: the noisy or heuristic decision-making people engage in when they find a problem too complex to solve precisely. This insight encourages further research aimed at tying together seemingly-distinct behavioral economics anomalies by focusing on the noise that is triggered by complexity. We believe a helpful tool in this regard will be to routinely measure cognitive uncertainty in experiments and surveys, especially given that it is fast and costless to do.

Our work relates to a growing interdisciplinary literature that documents that people often have an awareness of the noisiness of their choices, memories and perceptions, and that they take decisions that are in line with such awareness (e.g., Butler and Loomes, 2007; De Martino et al., 2013, 2017; Cubitt et al., 2015; Polania et al., 2019; Xiang et al., 2021; Drerup et al., 2017; Honig et al., 2020). Our main contribution to this literature is to document that cognitive uncertainty predicts biases across various economic decision tasks, and that it can be used to tie together anomalies that are typically viewed as distinct.

Our paper builds on a broad theoretical literature that has linked probability weighting and over- / underestimation of probabilities to different versions of noise. This includes the recent literature on Bayesian models of cognitive noise (Woodford, 2019; Gabaix, 2019; Gabaix and Laibson, 2017; Frydman and Jin, 2021), in particular the model of probability weighting in Khaw et al. (2021).[1] Other noisy decision models of probability weighting and over- / underestimation of probabilities include, for example, Bhatia (2014); Marchiori et al. (2015); Viscusi (1985, 1989); Blavatskyy (2007); Zhang et al. (2020); Erev et al. (1994).[2] Despite the abundance of such models, leading recent reviews rarely even mention a potential role of (cognitive) noise for the empirical regularities and instead emphasize models with fixed "probability weighting", "conservatism" or "extreme belief aversion" parameters that are partly even meant to capture preferences (e.g., Fehr-Duda and Epper, 2012; Benjamin, 2019). O'Donoghue and Somerville (2018) note that "the psychology of probability weighting is poorly understood." This view in the literature may reflect that few contributions directly measure noise or attempt to explain behaviors across different decision domains – both of which we contribute here.[3]

The paper proceeds as follows. Section 2 discusses theoretical background. Section 3 presents the experimental design. Sections 4–7 discuss the results and Section 8 concludes.

[1]Khaw et al. (2021) and Frydman and Jin (2021) also report experiments on cognitive noise and risk taking, but these do not test predictions related to probability weighting.

[2]Wakker (2010) likewise speculates that likelihood insensitivity in probability weighting reflects cognitive limitations. Erev et al. (2017) highlight how an "equal weighting" tendency leads to probability weighting.

[3]In a paper subsequent to ours, Oprea (2022) provides further evidence that probability weighting is driven by complexity by showing that the fourfold pattern of risk attitudes also holds when risk is removed from lottery choice problems. He reports that these patterns are strongly correlated with cognitive uncertainty.

2 Theoretical Considerations and Hypotheses

Various contributions have hypothesized that the patterns summarized in Figure 1 are driven by different types of noise. Khaw et al. (2021) model a decision-maker who exhibits cognitive noise when processing probabilities, which makes him regress towards an intermediate prior, hence producing probability weighting (also see Gabaix, 2019). Earlier related theoretical work modeled probability weighting as resulting from Bayesian updating from imperfect information about objective payout probabilities (Viscusi, 1989; Fennell and Baddeley, 2012), decision or sampling noise (Blavatskyy, 2007; Bhatia, 2014), affective vs. deliberate decision making (Mukherjee, 2010), or random fluctuations in risk preferences (Bhatia and Loomes, 2017). Similarly, multiple contributions have argued that regression of beliefs towards 50:50 may reflect noise or ignorance (Viscusi, 1985; Erev et al., 1994; Marchiori et al., 2015; Moore and Healy, 2008; Fischhoff and Bruine De Bruin, 1999).

Our analysis builds on these models. We here present a stylized adaptation that illustrates how we think about the commonalities reflected in Figure 1. Our exposition builds on the recent Bayesian cognitive noise literature (e.g., Khaw et al., 2021; Woodford, 2020; Heng et al., 2020), though our interpretation of these models is more agnostic.

Overview. We consider situations in which a decision-maker (DM) with Bernoulli utility function $u(\cdot)$ is tasked with making a decision a that depends on some objective probability p. We denote by $a^*(p) \in \underset{a}{argmax}\, EU(\cdot)$ the DM's true expected-utility maximizing decision. We assume that, through deliberation, the DM only has access to a noisy mental simulation of $a^*(p)$. The noisiness of this mental simulation may depend on the complexity of the decision problem.

Risky choice. The DM is asked to indicate his certainty equivalent for a lottery that pays \$1 with probability p and nothing otherwise. By standard arguments, normalizing $u(1) = 1$, the expected-utility maximizing decision is given by $a^* = u^{-1}(p)$.

Belief formation. In a fully controlled "balls-and-urns" belief updating task, the DM forms beliefs about a binary state of the world, R or B. The DM has prior $b = P(R)$ and receives a binary signal (H or L) with diagnosticity $h = P(H|R) = P(L|B)$. The Bayesian posterior belief is given by $p \equiv P(R|H) = P(B|L) = \frac{bh}{bh+(1-b)(1-h)}$. A widely-used formulation that we also leverage is a so-called Grether (1980) decomposition, which generates a linear relationship between the Bayesian posterior odds, the prior odds, and the likelihood ratio:

$ln(p/(1-p)) = ln(b/(1-b)) + ln(h/(1-h))$. We assume the incentive structure is such that it is optimal for the DM to report his true beliefs, such that the utility-maximizing decision is given by $a^* = p$.

Economic forecasts. The DM forecasts a future binary state of the world, R or B, that corresponds to a real economic quantity not controlled by the experimenter, such as inflation or stock market growth. In the past, the DM potentially received information about this state of the world, which he processes using Bayes' rule as described above, to arrive at posterior p. We again assume the incentive structure is such that it is optimal for the DM to report his true beliefs, $a^* = p$.

Bayesian cognitive noise. As noted above, we assume that the DM does not have access to the utility-maximizing decision $a^*(p)$. This could be due to a variety of reasons. In risky choice, the DM may not know his true utility function, may find it cognitively hard to integrate payoff probabilities and utils, or may have noisy perception. In laboratory belief updating tasks, the DM may not know Bayes' rule or struggle with implementing it computationally. In economic expectations surveys, the DM may have forgotten financial information that he received in the past, or he may struggle with processing the financial information available to him.

Whatever the underlying cognitive foundations, as we lay out formally in Appendix A, we assume that the DM has access to a cognitive signal S that is (scaled) Binomially distributed with precision N and satisfies $E[S] = a^*(p)$.[4] This cognitive signal could be interpreted as the outcome of a sequential cognitive sampling or deliberation process as in drift-diffusion models. Higher cognitive noise corresponds to a less precise Binomial signal. Relatedly, we can think of the level of cognitive noise – and, hence, the precision of the Binomial signal – as being determined by the complexity of the decision problem. Indeed, we will provide evidence below that higher complexity induces more cognitive noise.

Suppose that the DM holds a Beta-distributed prior over $a^*(p)$ and that his decision is given by the Bayesian posterior mean over his utility-maximizing decision.[5] We refer

[4]In contrast to Khaw et al. (2021), our framework features cognitive noise at the level of the utility-maximizing decision, rather than of a problem input parameter. We focus on noise in output space because we wish to be transparent that neither we nor our empirical cognitive uncertainty measure take a stance on what the source of cognitive noise is; we believe it is likely that there is more than one. Appendix A.4 discusses how similar predictions to the ones we state below emerge when one compares the behavior of a noiseless and a noisy agent in the framework of Khaw et al. (2021).

[5]This assumption has two different interpretations. A first one is that the DM chooses the posterior mean as a heuristic strategy. Indeed, it not entirely clear why a DM who cannot determine a Bayesian posterior

to the mean of the prior, d, as the "cognitive default decision." Given signal realization $s \sim f(s \mid a^*(p))$, a Bayesian DM's decision, a^o, can be represented as a convex combination of the cognitive signal and the prior mean, see Appendix A:

$$a^o = \lambda(N) \cdot s + [1 - \lambda(N)] \cdot d \tag{1}$$

$$E[a^o] = \lambda(N) \cdot a^*(p) + [1 - \lambda(N)] \cdot d \tag{2}$$

Here, the relative weight placed on the cognitive signal, $\lambda(N)$, increases in the signal's precision N. This decision rule is compatible with an anchoring-and-adjustment heuristic (Tversky and Kahneman, 1974), according to which people anchor on some initial reaction, d, and then adjust in the direction of the true utility-maximizing decision upon deliberation.

We interpret the prior mean d as the decision the DM would take in the absence of any deliberation. We do not provide a theory of what determines the prior. For our purposes, all that matters is that its mean is sufficiently "intermediate" in nature: for low enough p, $a^*(p) < d$, and for large enough p, $a^*(p) > d$. An intermediate prior implies that people's decisions look like they treat different payout probabilities as more similar than they really are, consistent with the emphasis on "equal weighting" in Erev et al. (2017). Indeed, a large literature argues that people's heuristic (or initial) responses to decision problems are intermediate, such as in research on central tendency effects in judgment and perception (e.g., Hollingworth, 1910; Petzschner et al., 2015; Xiang et al., 2021), compromise effects in choice (Simonson and Tversky, 1992; Beauchamp et al., 2019), and research that interprets 50:50 responses in economic expectations surveys as a manifestation of "I don't know" (Fischhoff and Bruine De Bruin, 1999). The prior distribution could also be partly adapted to which decision "makes sense" on average in a given context.[6]

Note that an alternative interpretation of the DM's decision process that is formally very similar to the Bayesian cognitive noise model in terms of its implications for observable decisions is that of random choice.[7]

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or a certainty equivalent should be cognitively capable of best-responding to relatively involved incentive structures. A second interpretation is that utility is linear and the DM best-responds to the incentives present in the experiment. In our belief updating experiments, the loss function is quadratic, such that the posterior mean is optimal. In our lottery choice experiments, the implied loss function under risk neutrality is the absolute distance, such that the median is optimal. However, with a Binomial distribution, using the mean instead of the median is without much loss because the mean of a Beta(a,b) variable is $a/(a+b)$, the mode is $(a-1)/(a+b-2)$ and the median lies between the two.

[6] In their study of inverse S-shaped probability and frequency estimates, Zhang and Maloney (2012) report that a 1/N formulation (N being the number of states of the world) captures people's cognitive anchor well.

[7] This second possible account of eq. (1) is that with probability λ the DM deliberates and plays his resulting cognitive signal s, whereas with probability $(1-\lambda)$ he plays randomly by drawing from a distribution function with mean d. Under this interpretation, the probability of playing randomly increases in the DM's

Discussion. The linear equation (2) corresponds to the widely-studied "neo-additive weighting function" that has attracted attention in the literature on choice under risk. Our stylized framework motivates this functional form by endogenizing its parameters: (i) the intercept increases in noise and (ii) the slope decreases in noise. A characteristic feature of this decision rule is the "flipping" property implied by Figure 1. For instance, in lottery valuation tasks, relative to a noiseless DM, a cognitively noisy DM is less risk averse for low payout probabilities yet more risk averse for high payout probabilities.

Equation (2) implies an attenuated but linear mapping between objective probabilities and decisions (when utility is linear). As summarized in Figure 1, decisions actually tend to be inverted S-shaped functions of objective probabilities. We explore how cognitive uncertainty relates to this phenomenon in Section 8 and Appendix D. To foreshadow this discussion, we find that empirically measured cognitive uncertainty is hump-shaped in objective probabilities, which helps understand why we typically observe a higher sensitivity of responses to probabilities close to the boundaries than at intermediate levels.

Predictions. Formal statements of predictions and proofs are relegated to Appendix A.

1. *Cognitive noise and compression effects.*
   (a) *In risky choice, cognitive noise is correlated with probability weighting: $\exists p^*$ such that, for $p < p^*$, certainty equivalents increase in cognitive noise and for $p > p^*$ they decrease in cognitive noise.*
   (b) *In stated beliefs and economic forecasts, cognitive noise is correlated with overestimation of small and underestimation of large probabilities. In Grether decompositions, cognitive noise is correlated with base rate insensitivity and conservatism.*
2. *The distance between the DM's decision and the utility-maximizing decision increases in cognitive noise.*

Empirical implementation: Cognitive uncertainty. People's actual level of cognitive noise is conventionally unobservable. To render the predictions testable, we make use of the idea that awareness of cognitive noise generates subjectively perceived uncertainty about what the utility-maximizing decision is. This *cognitive uncertainty* is measurable. In the context

cognitive noisiness. Note that the only difference between the Bayesian cognitive noise and random choice interpretations of equation (2) is whether the DM's average action is attenuated because he regresses to a fixed prior or because he chooses randomly. We embrace both of these interpretations.

of the framework sketched above, we define it as

$$p_{CU} \equiv P(|\ [a^*|S=s] - a^o\ | > \kappa). \tag{3}$$

Here, $a^*|S=s$ denotes the perceived posterior distribution about the maximizing decision, conditional on having received cognitive signal s. Intuitively, cognitive uncertainty captures the likelihood with which the DM thinks his utility-maximizing decision falls outside a window of arbitrary length κ around the decision that he actually chose.[8]

As we show in Appendix A, cognitive uncertainty decreases in the precision of the Binomial cognitive signal. This allows us to use cognitive uncertainty as a proxy for the magnitude of cognitive noise and, hence, λ. Our argument is not that awareness of cognitive noise necessarily causes the economic behavior of interest (though it may), but that it allows for the measurement of a concept that is difficult to quantify otherwise.

3 Experimental Design

3.1 Overview

As summarized in Table 1, we implemented two sets of experiments. The main set of experiments reported here, identified by letter *A*, was run in early 2022. Earlier experiments ("B") were run in 2019. We summarize both sets of experiments here but relegate a detailed exposition of the B experiments to Appendix E.

3.2 Decision tasks

Choice under risk. To estimate a probability weighting function, treatment *Risk A* elicited certainty equivalents for binary lotteries that paid $\$y \in \{15, 16, \ldots, 25\}$ with probability $p \in \{1, 5, 10, 25, 35, 50, 65, 75, 90, 95, 99\}$ percent, and nothing otherwise. Certainty equivalents were elicited using the BDM technique proposed by Healy (2018). Participants were instructed that for each lottery there is a list of questions that ask whether the participant prefers the lottery or a safe payment, where the safe payment increases as one goes down the list. Following Healy (2018), instead of asking participants to make a decision in every row of the list, we instructed them that they would tell us the safe amount at which they

[8] In empirical implementations, κ should be chosen so that the resulting measurement picks up as much variation as possible. This implies that the choice of κ depends on the response scale and should be neither too small nor too large, in order to avoid bunching at 1 or 0, respectively.

Table 1: Overview of experiments

Experiment	Components	# Particip.	Pool
Risk A	Baseline risky choice tasks (gains) Complex numbers manipulation	500	Prolific
Beliefs A	Baseline belief updating tasks Complex numbers manipulation	500	Prolific
Risk B	Baseline risky choice tasks (gains and losses) Compound lottery manipulation	700	AMT
Beliefs B	Baseline belief updating tasks Compound belief manipulation	700	AMT

Notes. All experiments elicited expectations about the one-year return of the S&P 500, and the B experiments additionally measured expectations about one-year inflation rates and the national income distribution. AMT stands for Amazon Mechanical Turk.

would switch from preferring the lottery to preferring the safe payment, and that we would then fill out the entire choice list based on their decision. Thus, participants simply entered a dollar amount into a text box to indicate their certainty equivalent, where entries were restricted to be between zero and the lottery upside. Each participant initially stated their valuation of six randomly selected lotteries.

The two main advantages of this design are that (i) it eliminates the need to go through a long choice list that may be mentally tiring for participants and (ii) it is well-known that the choice list procedure has its own effects on behavior (e.g., Beauchamp et al., 2019), and we wanted to ensure that our results on cognitive uncertainty do not just capture such choice list effects.

In treatment *Risk B*, on the other hand, we instead implemented standard choice lists of the type used by, for example, Tversky and Kahneman (1992); Bruhin et al. (2010); Bernheim and Sprenger (2019). The fact that the results turn out to be very similar suggests that the elicitation technique as such does not generate our results.

We often work with a simple linear transformation of elicited certainty equivalents, *normalized certainty equivalents*, which are given by the certainty equivalent divided by the upside of the lottery (a quantity that is by construction between 0 and 100%).

Belief updating. In designing a structured belief updating task, we follow the recent review by Benjamin (2019) and implement the workhorse paradigm of so-called "balls-and-urns" or "bookbags-and-pokerchips" experiments. In treatment *Beliefs A*, there are two bags, A and B. Both bags contain 100 balls, some of which are red and some of which are blue. The computer randomly selects one of the bags according to a pre-specified base rate. Subjects do not observe which bag was selected. Instead, the computer selects one or more balls from the selected bag at random (with replacement) and shows them to the subject. The subject is then asked to state a probabilistic guess that either bag was selected. We visualized this procedure for subjects using the image in Appendix Figure 8.

The three key parameters of this belief updating problem are: (i) the base rate $b \in \{1, 5, 10, 30, 50, 70, 90, 95, 99\}$ (in percent), which we operationalized as the number of cards out of 100 that had "bag A" as opposed to "bag B" written on them; (ii) the signal diagnosticity $d \in \{65, 75, 90\}$, which is given by the number of red balls in bag A and the number of blue balls in bag B (we only implemented symmetric signal structures such that $P(red|A) = P(blue|B)$); and (iii) the number of randomly drawn balls $M \in \{1, 3, 5\}$. These parameters were randomized across trials but always known to participants.

Each subject initially completed six belief updating tasks. Financial incentives were implemented through the binarized scoring rule (Hossain and Okui, 2013). Here, the probability of receiving a prize of \$10 was given by $\pi = \max\left\{0, 1 - 0.0001 \cdot (g - t)^2\right\}$, where g is the guess (in %) and t the true state (0 or 100).

Economic forecasts. All of our experiments also elicited forecasts of economic variables such as stock market returns. A conceptual difference between expectations about real-life quantities and the types of experimental tasks summarized above is that in the latter the experimenter supplies all information that the subject needs to make a well-defined rational decision, while in expectations surveys the experimenter does not have access to the respondent's information set. Still, cognitive uncrtainty can be measured in a very similar way. Indeed, intuitively, people may well exhibit cognitive uncertainty about their economic expectations: they may not perfectly remember their current beliefs about the stock market (or the information they received in the past), or they may worry that they have incorrectly processed past information.

In our A study ($N = 1,000$, see Table 1), we elicit probabilistic forecasts of the performance of the S&P 500. Because incentivizing expectations about future events creates various logistical issues such as credibility concerns and the necessity to wait for future variables to have realized, we elicited them without financial incentives. This is line with the vast majority of the literature on survey expectations. Each participant responsed to the following question:

> *The S&P 500 is an American stock market index that includes 500 of the largest companies based in the United States. Jon invested $100 in the S&P 500 today. What is the percent chance that the value of his investment will be less than $y in one year from now?*

Across participants, the value of y was drawn at random from the set {62, 77, 90, 100, 112, 123, 127, 131, 134}. These values were chosen such that the corresponding historical return probabilities (from 1980 to 2018) vary between 1% and 99%. For example, the historical probability that a $100 investment will be worth less than $127 one year later is 75%. In our "B" experiments, we also elicited beliefs about future inflation rates and the national income distribution in a very similar manner, see Appendix E.

3.3 Measuring Cognitive Uncertainty

Elicitation. In all decision tasks summarized above, decisions are given by a scalar. Loosely speaking, we always measure cognitive uncertainty (CU) on the subsequent screen by eliciting the participant's subjective probability that their expected-utility maximizing decision is contained in a window around their actual decision.

In choice under risk, we reminded participants of the lottery they were exposed to on the previous screen and then asked:

> *Your decision on the previous screen indicates that you value this lottery as much as receiving $x with certainty. How certain are you that you actually value this lottery somewhere between getting $(x-0.50) and $(x+0.50)?*

Participants answered this question by selecting a radio button between 0% and 100%, in steps of 5%. Appendix G.1 provides screenshots. In line with the discussion in Section 2, we interpret this question as capturing the participant's (posterior) uncertainty about their utility-maximizing decision, after some sampling of cognitive signals has taken place. We refer to inverted responses to this question as *cognitive uncertainty* rather than confidence because in economics the latter is commonly used for problems that have an objectively correct solution.

In belief updating, the instructions introduced the concept of an "optimal guess." This guess, we explained, uses the laws of probability to compute a statistically correct statement of the probability that either bag was drawn, based on Bayes' rule. We highlighted that this optimal guess does not rely on information that the subject does not have. After indicating their probabilistic belief, subjects were asked (see Appendix Figure G.2):

Your decision on the previous screen indicates that that you believe there is a x% chance that Bag A was selected. How certain are you that the optimal guess is somewhere between (x-1)% and (x+1)%?

In economic forecasts, the elicitation is very similar, asking how certain the respondent is that their probabilistic guess is within a one percentage point band around the guess that's optimal *given the information available to the respondent*. Thus, the question does not elicit people's awareness of their lack of information, but instead their perceived ability to appropriately remember and process the information available to them (see Appendix Figure G.3):

On the previous screen, you indicated that you think there is a x% chance that a $100 investment into the S&P 500 today will be worth less than $y in one year from now. How certain are you that the statistically optimal guess (given the information you have) is somewhere between getting (x-1)% and (x+1)%?

The biggest difference between our A experiments and the B experiments conducted earlier is the wording of the CU question. In the B experiments, we did not elicit participant's subjective probability that the utility-maximizing decision is within some fixed band around their actual decision, but rather a heuristic confidence interval. In choice under risk, subjects used a slider to calibrate the statement "I am certain that the lottery is worth between a and b to me." If the participant moved the slider to the very right, a and b corresponded to the previously indicated certainty equivalent. For each of the 20 possible ticks that the slider was moved to the left, a decreased and b increased by 25 cents, in real time. In belief updating questions and economic forecasts, subjects navigated a slider to calibrate the statement "I am certain that the optimal guess [*economic forecasts:* statistically optimal guess] is between a and b.", where a and b collapsed to the subject's own previously indicated guess in case the slider was moved to the very right. For each of the 30 possible ticks that the slider was moved to the left, a decreased and b increased by one percentage point. See Appendix E for further details. We believe the new measure to be superior in that it admits a direct quantitative interpretation and is more intuitive for subjects. This being said, the results are qualitatively very similar across both sets of experiments.

Potential origins of cognitive uncertainty. Our measure is deliberately designed to capture participants' overall subjective uncertainty about what their utility-maximizing decision is. This uncertainty could have various potential origins. In choice under risk, people may have imperfect perception, may not know their true preferences, or struggle with integrating utils and probabilities. In belief updating, participants may not know the normatively correct updating rule, or struggle with its computational implementation. In survey expectations, they may not remember information they received in the past, or may again implement an incorrect updating rule. While we conjecture that it will often be of secondary interest to economists what exactly the source of cognitive noise is (there are likely many), we caution that our measure does not allow researchers to directly test models that take a direct stance on the source of the noise.

Comparison with alternative measures. Broadly speaking, the literature has proposed two different types of measures for eliciting people's uncertainty about their own decisions. At one extreme, psychologists, neuroscientists and some economists elicit measures of "decision confidence," in which subjects indicate on Likert scales how confident or certain they are in their decision (e.g., De Martino et al., 2013, 2017; Polania et al., 2019; Xiang et al., 2021; Butler and Loomes, 2007; Drerup et al., 2017). At the other extreme, economists have used measures of across-trial variability in choices (Khaw et al., 2021) or deliberate randomization (Agranov and Ortoleva, 2017, 2020). Our preferred measure strikes a middle ground between these two approaches. While our approach retains the attractive simplicity of implementing a single question (as in the psychology literature), it is also admits a direct quantitative interpretation in terms of a subjective percent chance.[9] The simplicity of asking one question per decision should be contrasted with the approach of gauging cognitive noise through across-task variability in choices, which requires *many* trials and is often defined at the level of a study rather than of a single choice problem.

Financial incentives and validation. We deliberately do not financially incentivize our elicitation of CU, for two reasons. First, an additional scoring rule makes the measure itself more complex, which increases the cognitive burden on participants. Indeed, recent work documents that unincentivized measures of beliefs are sometimes superior to incentivized ones because they reduce the strategic incentives to game a potentially complex (and misperceived) scoring rule (Danz et al., 2020). Second, we believe that financially incentivizing the measurement in potentially complicated ways would increase the costs for future researchers to include a CU measure in their experiments and surveys.

We validate our simple-but-unincentivized measure below by documenting correlations

[9]We have found that economists are often more comfortable with uncertainty questions that have a precise quantitative meaning in terms of probabilities, which Likert scales do not.

with across-trial variability in repetitions of the same decision problem, which is commonly viewed as a key signature of cognitive noise.

3.4 Complexity Manipulations

Our experiments link cognitive noise to decisions in two ways. First, we correlate decisions with cognitive uncertainty (awareness of noise). Second, we exogenously manipulate the noisiness of decisions by making the decision tasks more complex. In doing so, we focus on the choice under risk and balls-and-urns belief updating experiments because they allow for more controlled variation.

Complex numbers. In our main experiments, *Risk A* and *Beliefs A*, the complexity manipulation is given by representing payout probabilities (in choice under risk) and base rates / signal diagnosticies (in belief updating) as mathematical expressions, such as "Get \$20 with probability $(7 \times 6/2 - 11)$%." These treatments were implemented in a between-subjects design: after each subject had completed six baseline tasks of either risky choice or belief updating, for a second set of six tasks they were randomized into either another set of baseline tasks or a set of the complex numbers tasks.

Compound problems. In our experiments *Risk B* and *Beliefs B*, we instead manipulated complexity by deploying compound problems. We hypothesize that these are more complex for people to think through than the normatively identical reduced problems. The compound problems were randomly interspersed with the respective baseline problems in a within-subjects design. In choice under risk, if a baseline lottery is given by a p% chance of getting \$20, then the corresponding compound lottery is to get \$20 with probability $p' \sim U\{p - 0.05, \ldots, p + 0.05\}$. In terms of implementation, we told participants that the probability of receiving the lottery upside was unknown to them and would be randomly determined by drawing from a known interval, such that each integer is equally likely to get drawn. Because expected utility is linear in probabilities, this compound manipulation does not affect the normative benchmark for behavior.

In belief updating, if a baseline updating problem features signal diagnosticity h and base rate $b = 50\%$, then the corresponding compound updating problem features diagnosticity $h' \sim U\{h-0.1, \ldots, h+0.1\}$. It is straightforward to verify that the Bayesian posterior for these two updating problems is identical.

3.5 Experiments A and B

We here summarize the main differences between treatments *Risk A* and *Beliefs A* on the one hand, and *Risk B* and *Beliefs B* on the other hand. (i) The CU measurement differs in wording and quantitative interpretation. (ii) The risky choice tasks were implemented using different procedures: with a BDM mechanism à la Healy (2018) in the A experiments and as a visual multiple price list in the B experiments. (iii) The complexity manipulations differ. Moreover, these were implemented in a between-subjects format in the A experiments and in a within-subjects format in the B experiments. (iv) The A experiments feature some repeated, identical problems that allow us to study choice variability. (iv) The B experiments include a broader set of questions measuring economic forecasts.

3.6 Logistics and Participant Pool

As summarized in Table 1, our A experiments were conducted on Prolific, while the B experiments were run on Amazon Mechanical Turk. The B experiments were pre-registered, see Appendix E.

In both sets of experiments, we took two measures to achieve high data quality. First, our financial incentives are unusually large both by AMT and Prolific standards. Average hourly earnings in our experiments exceed the target compensation on those platforms by roughly 190% and 250%, respectively. Second, we screened out inattentive prospective subjects through comprehension questions and attention checks. In total, 53% and 54% of all prospective participants were screened out in experiments *Risk* and *Beliefs*, respectively. Screenshots of instructions and comprehension check questions can be found in Appendix G.

The timeline of *Risk A* and *Beliefs A* was as follows: (i) main incentivized task; (ii) hypothetical economic forecast question; (iii) incentivized Raven matrices test; (iv) demographic questionnaire. Participants received a completion fee of $3 in both treatments. In addition, each participant potentially earned a bonus. With probability 30%, a randomly-selected task of part (i) was payoff-relevant and with probability 70% part (iii) was paid out. Average earnings in *Risk A* were $8.10 and $4.80 in *Beliefs A*.

4 Cognitive Uncertainty: Variation and Validation

Variation. Figure 2 shows histograms of task-level CU in the baseline tasks of *Risk A* and *Beliefs A* as well as for stock market expectations. The magnitude of CU should not be compared across decision domains because the length of the interval with respect to which CU

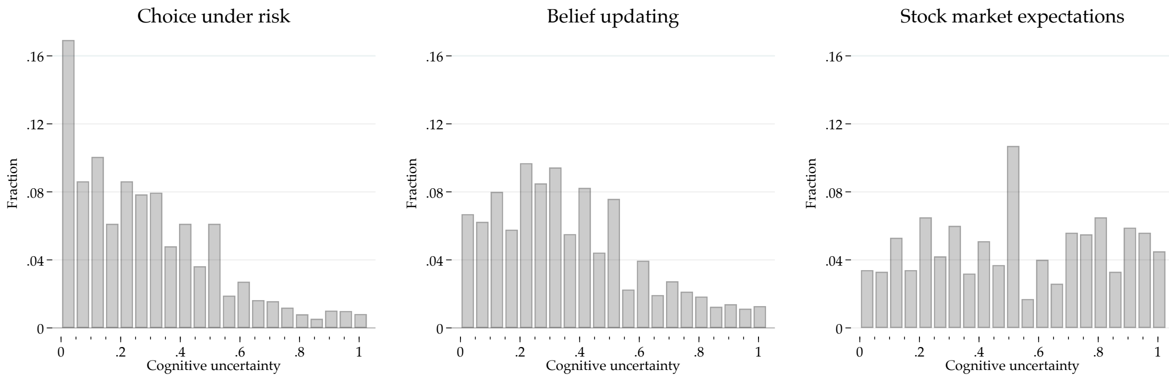


Figure 2: Histograms of cognitive uncertainty in the baseline tasks in *Risk A* ($N = 4,524$), *Beliefs A* ($N = 4,590$) and stock market expectations ($N = 1,000$).

is measured is not comparable. Rather, we show these histograms side-by-side to illustrate (i) that a large majority of decisions reflect strictly positive CU and (ii) the large heterogeneity in CU. 83% of the certainty equivalents in *Risk A*, 93% of beliefs in *Beliefs A* and 97% of stock market forecasts are associated with strictly positive CU.

Stability. An obvious question is whether the unincentivized CU question picks up real variation or just noise. A first indication is to look at whether the histograms shown above largely capture within- or across-subject variation. In lottery choice and belief updating, where we observe multiple decisions per subject, 51–54% of the variation in the CU data is explained by participant fixed effects. Given that some of the residual variation likely reflects measurement error, this suggests that across-subject variation is the dominant source of variation in the CU data, and that participants are relatively consistent in their degree of CU within a given domain.

A second indicator for stability is a within-subject test-retest correlation. This is feasible in our context because in lottery choice and belief updating we implemented at least two decision problems twice. We find that CU is highly correlated across these randomly interspersed elicitations ($r = 0.70$ in *Risk* and $r = 0.68$ in *Beliefs*).

A final indicator for stability is cross-domain stability. We correlate average CU in choice under risk with CU in stock market expectations, and average CU in lab beliefs with CU in stock market expectations. The Spearman correlations are given by $\rho = 0.19$ for risky choice and $\rho = 0.35$ for belief updating ($p < 0.01$ for both correlations). This further suggests some within-person stability of CU.

Table 2: Correlates of cognitive uncertainty

	Dependent variable: Cognitive uncertainty					
	Choice under risk		Belief updating		Stock market exp.	
	(1)	(2)	(3)	(4)	(5)	(6)
1 if female	0.064***	0.063***	0.054***	0.052***	0.11***	0.11***
	(0.02)	(0.02)	(0.01)	(0.01)	(0.02)	(0.02)
Age	-0.0017***	-0.0017***	-0.0016***	-0.0016***	-0.0048***	-0.0048***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Ln [Time taken for study]	-0.018	-0.018	0.011	0.011	0.034	0.034
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
Raven matrices score (0-4)	-0.022**	-0.021**	-0.00076	-0.0014	0.0035*	0.0035*
	(0.01)	(0.01)	(0.01)	(0.01)	(0.00)	(0.00)
1 if college degree	0.00059	0.0023	0.023	0.024	-0.029	-0.029
	(0.02)	(0.02)	(0.01)	(0.01)	(0.02)	(0.02)
Extremity of normative decision		-0.19***		-0.31***		0.00026
		(0.03)		(0.03)		(0.00)
Constant	0.47***	0.52***	0.28**	0.37***	0.36**	0.35**
	(0.14)	(0.14)	(0.12)	(0.12)	(0.16)	(0.16)
Observations	4524	4524	4602	4602	1000	1000
R^2	0.04	0.05	0.02	0.06	0.09	0.09

Notes. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. The extremity of the normative decision is given by the absolute distance of the normative decision to 50%, where the normative decision is assumed to reflect risk-neurality in lottery choice, Bayesian beliefs in belief updating and historical probabilities in stock market expectations. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Correlates. Regarding demographic correlates of CU, the most consistent pattern is that – across all three decision domains – women report about 5-11 percentage points higher CU, akin to a large body of evidence on other domains of confidence (see Table 2). We also find that older participants report lower CU, though the quantitative magnitude of this relationship is small. Meanwhile, total response time for the survey and proxies for cognitive ability (score on a Raven matrices test and a college degree) are largely unrelated to CU.

Finally, in lottery choice and belief updating, CU strongly decreases in the extremity of the normative benchmark, i.e., the absolute distance of the normative benchmark to 50%. In lottery choice, subjects indicate lower CU if the payout probability is far away from 50%, suggesting that, for example, valuing a lottery with payout probability of 95% is easier than valuing a lottery with payout probability 60%. In belief updating, CU reveals that subjects find it easier to state beliefs for problems that have Bayesian posteriors close to zero or one.

Cognitive uncertainty and choice variability. Some researchers have used choice variability as an empirical measure of cognitive noise (e.g., Khaw et al., 2021). We examine the empirical correspondence between our CU question and variability for two reasons. First, data on choice variability is useful to understand whether people's subjective perception of their own noisiness is roughly accurate. Second, a correlation between CU and choice variability may be seen as validation of our quantitative-but-unincentivized question, in the spirit of recent experimental validation studies in the literature (e.g., Falk et al., 2015).

We compute across-trial variability as absolute difference in decisions across two repetitions of the same problem. We find that decisions that are associated with higher average CU across the two trials are more variable, see Appendix Figure 6. The Spearman correlation is $\rho = 0.27$ in choice under risk and $\rho = 0.30$ in belief updating ($p < 0.01$ in both datasets). These results resonate with those from our work on cognitive uncertainty in intertemporal choice, in which cognitive uncertainty and across-trial variability in responses are likewise significantly correlated (Enke et al., 2022).

5 Results: Cognitive Uncertainty Predicts Bias

5.1 Visual Illustration of Compression Effects

We begin by analyzing the data in the baseline tasks.[10] The left-hand panels of Figure 3 summarize the link between cognitive uncertainty and compression effects in the treatment of probabilities. Both panels are constructed following the same logic, by plotting participants' (normalized) decisions against objective probabilities. Panel A shows normalized certainty equivalents as a function of payout probabilities in *Risk A*. Panel B shows posterior beliefs as a function of Bayesian posteriors in *Beliefs A*. Panel C shows subjective stock return expectations as a function of historical probabilities. The dots show medians within the samples of above- and below-median cognitive uncertainty decisions, respectively.

We see that decisions are always substantially more compressed towards intermediate options in the presence of higher cognitive uncertainty. For instance, in choice under risk, the median decision of low cognitive uncertainty subjects is frequently visually indistinguishable from the benchmark of no probability weighting. This pattern implies the "flipping" property discussed in the theoretical framework: cognitively uncertain decisions are

[10]In both *Risk A* and *Beliefs A*, each subject first completed six such baseline tasks, after which half the subjects completed six additional baseline tasks, while the remaining half completed the complex math problems. As a result, the data in this section consist of twelve tasks for some subjects and six tasks for others.

less risk averse for low probabilities but more risk averse for high probabilities. We interpret these patterns as showing that what the literature often refers to as "probability-dependent risk preferences" are, in fact, due to bounded rationality (cognitive noise).

It is instructive to compare the patterns in Panel A with those that should be expected from an expected-utility maximizer. As discussed in Section 2, normalizing utility from the lottery upside to one, the expected-utility maximizing decision is given by $a^* = u^{-1}(p)$. Under risk neutrality, normalized certainty equivalents should be located on the 45-degree line. Under strict risk aversion, they should be a convex increasing function of payout probabilities, located strictly below the 45-degree line.

In the belief updating task, Panel B, the median posteriors of low cognitive uncertainty decisions are likewise relatively close to the rational benchmark. In contrast, cognitively uncertain beliefs reflect pronounced overestimation of small and underestimation of high probabilities. Thus, the phenomenon of "extreme belief aversion" discussed in the review by Benjamin (2019) reflects cognitive noise rather than preferences.

For the stock market expectations data, Panel C, we plot participants' answers against corresponding historical probabilities. Recall that participants never saw these probabilities – we imputed them from the values of the returns whose probability the participants were asked to assess. Similarly to the lab belief updating task, we see that cognitive uncertainty is predictive of overestimation of small and underestimation of large probabilities.

The right-hand panels of Figure 3 provide a more complete picture of the relationship between cognitive uncertainty and sensitivity to objective probabilities. We now split the sample into cognitive uncertainty quartiles. Because in our lottery choice and belief updating experiments between 20% and 25% of all CU statements are equal to zero, the first quartile in these two experiments almost corresponds to $CU = 0$, while the other quartiles leverage variation in the intensive margin of CU. For each of the four CU buckets, we regress observed (normalized) decisions on the respective objective probability (payout probability in choice under risk, Bayesian posterior in belief updating and historical probability in stock market expectations), and report the coefficient. If decisions did not depend on cognitive noise, the four regression coefficients would be equally large. Instead, we see that the effect of objective probabilities monotonically decreases as CU increases. This shows that the results are not just driven by the extensive margin of CU, but that higher CU is strongly associated with more compression also within the sample of strictly positive CU.

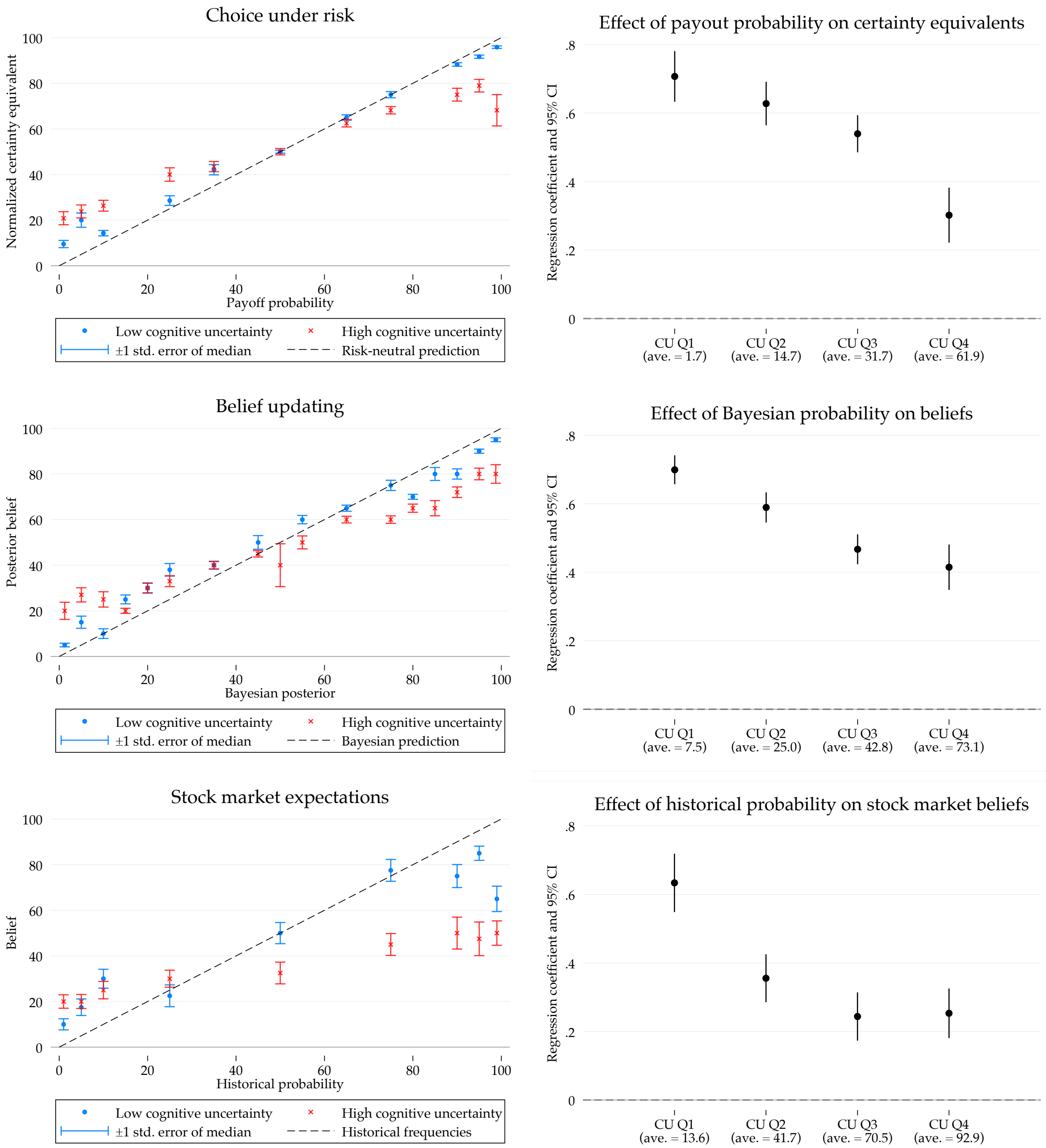


Figure 3: Left panels: median normalized certainty equivalents as function of payout probabilities (top, *Risk A*), median beliefs as function of binned Bayesian posteriors (middle, *Beliefs A*) and median stock market expectations as function of historical probabilities (bottom). All panels display bins with 30 or more observations. Low CU is below median. Whiskers show standard error bars. Right panels: coefficients from OLS regressions of (normalized) decisions on objective probabilities, split by CU quartiles. Effect of payout probability on stated certainty equivalents (top, *Risk A*), effect of Bayesian posterior on stated beliefs (middle, *Beliefs A*) and effect of historical probability on stated stock market expectations (bottom). Whiskers show 95% confidence intervals.

5.2 Regression Evidence

5.2.1 Choice under risk

Table 3 studies the link between CU and likelihood insensitivity (probability weighting) in risky choice more formally, through regression analyses. We estimate the neo-additive weighting function in eq. (2). To this effect, we regress certainty equivalents on the payout probability, cognitive uncertainty and their interaction. The framework in Section 2 predicts that (i) the interaction coefficient is negative (indicating a shallower slope) and (ii) the raw cognitive uncertainty term is positive, indicating a higher intercept.

Columns (1)–(2) of Table 3 document that both of these predictions are indeed borne out in the data. In quantitative terms, an increase in cognitive uncertainty from 0% to 50% is associated with a decrease in the slope of certainty equivalents with respect to payout probabilities by 33.5 percentage points, a very large magnitude.

We likewise find that cognitive uncertainty is strongly related to the regression intercept, as predicted by the model. In other words, the positive cognitive uncertainty raw term does not mean that the probability weighting function of cognitively uncertain subjects has higher elevation on average – it just means that the elevation at $p = 0$ is higher.

Columns (3)–(6) provide further evidence that these patterns imply the characteristic "flipping" pattern that we anticipated in the discussion of the theoretical framework: for small probabilities, cognitively uncertain decisions reflect significantly *more* risk seeking, while for high probabilities they reflect less risk seeking.

Losses and MPL elicitation technique. Our earlier B experiments allow us to probe the robustness of our results along two dimensions. First, we studied both gain and loss lotteries. Second, the certainty equivalents were elicited using standard visual multiple price lists. The results in these experiments are very similar to those reported above, in the sense that cognitively uncertain decisions are significantly more compressed. This is true for both gains and losses, see Appendix E.

The results in the B study imply a nuanced pattern about how CU is correlated with risk seeking vs. risk averse behavior. Because CU is associated with "overweighting" of small and "underweighting" of large probabilities for both gains and losses, we have that high CU decisions reflect risk-seeking behavior for low probability gains and high probability losses, but risk-averse behavior for high probability gains and low probability losses. In other words, CU is predictive of the so-called fourfold pattern of risk attitudes.

Table 3: Cognitive uncertainty and likelihood insensitivity in *Risk A*

	Dependent variable: Normalized certainty equivalent					
	Full sample		$p < 50\%$		$p \geq 50\%$	
	(1)	(2)	(3)	(4)	(5)	(6)
Payout probability	0.73***	0.73***	0.56***	0.55***	0.60***	0.59***
	(0.03)	(0.03)	(0.06)	(0.05)	(0.04)	(0.04)
Payout probability × Cognitive uncertainty	-0.67***	-0.67***				
	(0.08)	(0.08)				
Cognitive Uncertainty	25.1***	22.7***	15.0***	11.0**	-26.3***	-27.0***
	(6.18)	(6.02)	(5.64)	(5.48)	(3.51)	(3.60)
Constant	19.7***	31.5***	22.3***	39.4***	30.7***	38.1***
	(2.35)	(4.38)	(2.35)	(6.09)	(3.23)	(4.66)
Demographic controls	No	Yes	No	Yes	No	Yes
Observations	4524	4524	2035	2035	2489	2489
R^2	0.49	0.50	0.10	0.15	0.32	0.32

Notes. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. Demographic controls include age, gender, college education and performance on a Raven matrices test. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

5.2.2 Belief Updating

Table 4 studies the link between cognitive uncertainty and belief updating in *Beliefs A*. Again, the framework predicts that cognitive uncertainty should be related to (i) lower sensitivity of beliefs to variation in objective probabilities and (ii) a higher intercept. Columns (1)–(2) directly estimate the neo-additive decision rule in eq. (2) that our framework motivates. Here, we link observed beliefs to Bayesian posteriors, cognitive uncertainty and their interaction. Consistent with the visual impression from the left-hand panels of Figure 3, cognitively uncertain beliefs are substantially less sensitive to variation in Bayesian posteriors, and their intercept is higher. In terms of quantitative magnitude, the regression coefficients imply that moving from cognitive uncertainty of 0% to 50% is associated with a decrease of the slope by 21 percentage points.

Grether regressions: Inelasticity to base rate and likelihood ratio (conservatism). The literature typically highlights not only deviations of stated from Bayesian beliefs, but also the ways in which people implicitly respond to variation in the base rate, the likelihood ratio and the sample size (see Benjamin, 2019, for a review). As discussed in Section 2, we are

Table 4: Cognitive uncertainty and belief updating in *Beliefs A*

	Dependent variable:					
	Posterior belief		Ln [Posterior odds]			
	(1)	(2)	(3)	(4)	(5)	(6)
Bayesian posterior	0.71***	0.71***				
	(0.02)	(0.02)				
Bayesian posterior × Cognitive uncertainty	-0.43***	-0.43***				
	(0.06)	(0.06)				
Cognitive Uncertainty	12.8***	12.7***	-0.47***	-0.49***	-0.48***	-0.49***
	(3.54)	(3.53)	(0.14)	(0.14)	(0.13)	(0.14)
Ln [Bayesian odds]			0.55***	0.55***		
			(0.02)	(0.02)		
Ln [Bayesian odds] × Cognitive uncertainty			-0.42***	-0.42***		
			(0.07)	(0.07)		
Log[Prior Odds]					0.69***	0.69***
					(0.03)	(0.03)
Log[Likelihood Ratio]					0.37***	0.37***
					(0.03)	(0.03)
Ln [Prior odds] × Cognitive uncertainty					-0.52***	-0.52***
					(0.10)	(0.10)
Ln [Likelihood ratio] × Cognitive uncertainty					-0.21***	-0.21***
					(0.07)	(0.07)
Constant	19.5***	18.7***	0.23***	0.29**	0.24***	0.28**
	(1.53)	(2.22)	(0.06)	(0.12)	(0.06)	(0.11)
Demographic controls	No	Yes	No	Yes	No	Yes
Observations	4602	4602	4602	4602	4602	4602
R^2	0.49	0.49	0.45	0.45	0.48	0.48

Notes. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. To avoid a mechanical loss of observations resulting from the log odds definition, the log posterior odds in columns (3)–(6) are computed by replacing stated posterior beliefs of 100% and 0% by 99% and 1%, respectively. The results are virtually identical without this replacement. Demographic controls include age, gender, college education and performance on a Raven matrices test. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

interested in whether cognitive noise could generate the well-known phenomena of base rate insensitivity, conservatism (likelihood ratio insensitivity) and sample size insensitivity.

To analyze this empirically, we resort to so-called Grether regressions (Grether, 1980). This specification is derived by expressing Bayes' rule in logarithmic form, which implies a linear relationship between the posterior odds, the prior odds, and the likelihood ratio. The canonical finding in the literature is that in these regressions the observed coefficients of the log prior odds and the log likelihood ratio are usually considerably smaller than the Bayesian benchmark of one. As discussed in Section 2 and shown in Appendix A, our stylized cognitive noise model predicts that higher cognitive noise leads to higher insensitivity in these regressions. A simple intuition is that if someone always stated posterior beliefs of 50:50, their sensitivity of beliefs to the base rate and likelihood ratio would be zero.

Columns (3) and (4) of Table 4 estimate a restricted version of a Grether regression, in which we relate the subject's log posterior odds to the log Bayesian odds. This analysis is instructive because it takes place in log odds space (as motivated by the Grether decomposition), but essentially uses the same variables as in columns (1) and (2). Again, we find that cognitive uncertainty is strongly predictive of the degree of insensitivity of log posterior odds with respect to the Bayesian benchmark.

Finally, columns (5) and (6) estimate a standard Grether regression, except that we also account for interactions with cognitive uncertainty. The negative interaction coefficients show that cognitive uncertainty is strongly related to base rate insensitivity and likelihood insensitivity (conservatism). The quantitative magnitudes of the regression coefficients suggest that, for example, base rate sensitivity decreases from 0.69 with CU of 0% to 0.43 with CU of 50%.[11] These patterns document that (at least a part of) what this literature has identified as base rate neglect, conservatism and extreme belief aversion are in fact not independent psychological phenomena but instead all generated by cognitive noise and resulting compression effects.

Sample size effects. As is well known in the literature, experimental data also reveal systematic variation in stated beliefs *conditional* on Bayesian posteriors. For instance, for a given base rate, the draw of one blue ball gives rise to the same Bayesian posterior as the draw of two blue balls and one red ball, yet experimental participants consistently update more strongly after observing one blue ball (Benjamin, 2019). A common explanation is that subjects update based on sample *proportions*, while Bayesian updating prescribes updating based on sample *differences*. Our account of cognitive uncertainty also provides an explanation for this pattern. The straightforward reason is that stated cognitive uncertainty significantly increases in the sample size, holding the sample difference and the Bayesian posterior fixed (see Appendix Table 6). That is, subjects appear to find it easier to form beliefs based on one blue ball than based on two blue balls and one red ball. As a result of this systematic variation in cognitive noise, our account correctly predicts that subjects respond more to the sample difference when the sample size is smaller.

[11]The interaction coefficients are larger for the log prior odds than for the log likelihood ratio. We can only speculate about why this is the case. In our experiment, base rates are displayed using sets of cards, while diagnosticities are displayed using urns that are filled with 100 colored balls. We cannot rule out that this difference in the way in which information is presented affects the perceived complexity of these decision parameters and / or their interaction with cognitive noise.

Earlier experiments. All of the patterns summarized above also hold in our earlier B experiments, see Appendix E.

5.2.3 Stock market expectations

Appendix Table 9 presents regression analyses that confirm the visual impression from Figure 3: cognitive uncertainty is strongly predictive of the degree to which historical stock returns map into probabilistic forecasts. In our earlier B experiments, we find almost identical patterns for the same measure of stock market expectations. Moreover, we find very similar patterns of cognitive uncertainty predicting compression towards 50:50 also for inflation expectations and beliefs about the income distribution. See Appendix E.

5.3 Cognitive Uncertainty and Distance to the Optimal Decision

Thus far, the analyses documented that *average* decisions are more compressed and further away from normative benchmarks when they are associated with higher cognitive uncertainty. In itself, however, this does not imply that cognitively uncertain decisions are located further away from normative benchmarks, on average. To see this, consider a simple example in which the normatively optimal posterior in a belief updating task is 80%. Then, the average of stated beliefs of 79% and 77% is located further away from the normative benchmark than the average of beliefs of 60% and 100%, yet the average absolute distance is still smaller in the former case.

Our stylized model predics that cognitive noise produces not only stronger compression of the average, but also that it leads to larger average absolute distances to the normatively optimal decision. We here test this additional prediction. For belief updating, we use the Bayesian posterior as the normative benchmark. For survey expectations, we use historical probabilities. For choice under risk, we assume that subjects' objective is to maximize expected value. However, we have verified that very similar results hold when we infer the "true" utility-maximizing decisions by estimating a population-level CRRA parameter.

Figure 4 summarizes the results. Cognitive uncertainty and absolute distances to the normative benchmark are significantly correlated (Spearman's $\rho = 0.31$ in risky choice, $\rho = 0.17$ in beliefs and $\rho = 0.21$ in stock market expectations, $p < 0.01$ for all comparisons).

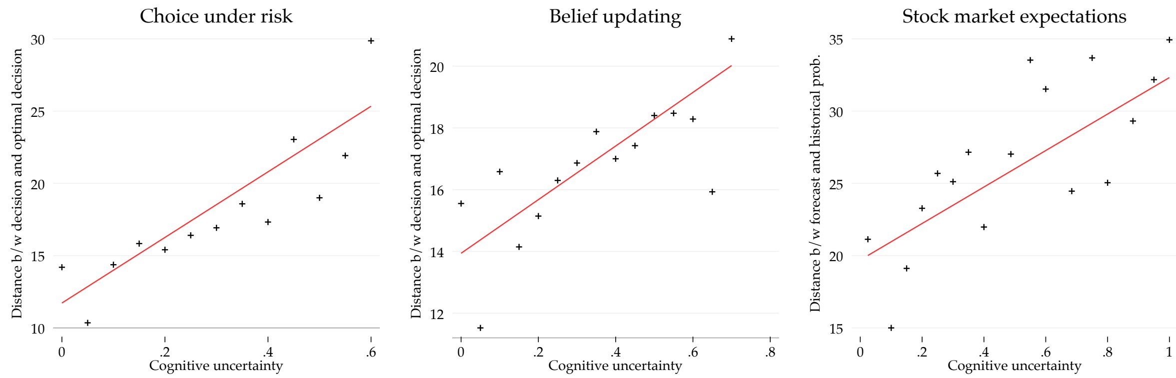


Figure 4: Binscatter plots of absolute distance between decisions and "normatively optimal" decisions as a function of cognitive uncertainty (*A* experiments). In the left panel ($N = 4,524$), the normative benchmark is assumed to be expected value maximization, in the middle panel ($N = 4,590$) it is the Bayesian posterior and in the right panel ($N = 1,000$) it is historical probabilities. Cognitive uncertainty is winsorized at the 90th percentile in each dataset.

5.4 Measurement Error in Cognitive Uncertainty

A prominent concern regarding the measurement of cognitive or preference constructs in experiments is measurement error (Gillen et al., 2019). In our context, measurement error in the CU elicitation could have two implications. First, CU and certainty equivalents / beliefs could be subject to a form of correlated measurement error that would potentially create a mechanical relationship between the occurrence of strictly positive CU and the sensitivity of decisions to objective probabilities. To illustrate, suppose that all subjects actually exhibit zero cognitive noise. Further suppose that (i) more inattentive subjects are more likely to exhibit random measurement error in the CU elicitation that leads them to state strictly positive CU, and (ii) that this same inattention will also lead subjects to state risky decisions or beliefs that are insensitive to objective probabilities. Under this logic, CU and observed decisions would be mechanically correlated. If this were the case, however, we would expect that CU has no predictive power for decisions within the sample of strictly positive CU. As the right-hand panels of Figure 3 showed, this is counterfactual as the sensitivity of decisions to delays strongly decreases in CU, even conditional on CU > 0.

A second implication of measurement error in CU could be coefficient attenuation. A standard remedy against this is to instrument out measurement error through repeated elicitations (Gillen et al., 2019). This is feasible in our data because every subject completed at least two decisions twice. As noted above, cognitive uncertainty is highly correlated across these repetitions of the same decision problem ($r = 0.70$ in *Risk A* and $r = 0.68$ in *Beliefs A*). This enables "Obviously Related Instrumental Variable" (ORIV) analyses, see Appendix

Tables 7 and 8. Here, we replicate our OLS regressions from Tables 3 and 4, except that we instrument for the interaction between objective probabilities and CU with the interaction between objective probabilities and CU from the repeated elicitation. The results are almost identical. This suggests that measurement error in the CU elicitation is not a major concern.

6 Complexity, Cognitive Noise and Compression Effects

In the conceptual framework in Section 2, we took the magnitude of cognitive noise (captured by N) as given. More realistically, cognitive noise will be higher if the complexity of a decision problem is high. As outlined in Section 3, our A experiments manipulated problem complexity by expressing probabilities as math problems. The B experiments instead manipulated complexity through compound problems.

Given that there is no widely accepted theory of what is (not) complex, neither of these two treatments is directly theoretically motivated. However, multiple previous contributions have hypothesized that compound problems or complex numbers can make decision problems harder (e.g. Huck and Weizsäcker, 1999; Gillen et al., 2019). Moreover, an added benefit of our cognitive uncertainty measurement is that it allows us to directly test whether a complexity intervention actually increases cognitive noise. Both experimental manipulations had large effects on cognitive uncertainty. The complex numbers manipulation increased CU by 45% in risky choice and by 48% in belief updating. The compound manipulations lead to an increase in CU by 23% in risky choice and by 33% in belief updating.[12]

Panels A–D of Figure 5 document that this increase in complexity (and resulting cognitive noise) has a large effect on decisions. As predicted, responses are always substantially more compressed towards an intermediate value than in our baseline experiments. This is true for both the math manipulation and the compound problems.[13] Appendix Tables 10–13 provide corroborating regression evidence.[14] Overall, we interpret these patterns as evidence that cognitive noise actually causes compression towards an intermediate value, rather than that it only correlates with it.

We also note that all of these results are inconsistent with a large class of models of probability weighting and belief updating biases that rest on the assumption of fixed para-

[12]Recall that we used a different CU measure in the B experiments, such that the magnitudes of the CU increase should not be directly compared across experiments.

[13]It is interesting to relate these results to Harbaugh et al. (2010). They identify evidence for probability weighting in one elicitation mechanism but not another one, and interpret this by suggesting that the mechanism that produces probability weighting is "more complex."

[14]In experiment *Risk B*, we also implemented compound lotteries for loss gambles. The results are very similar, see Appendix E.

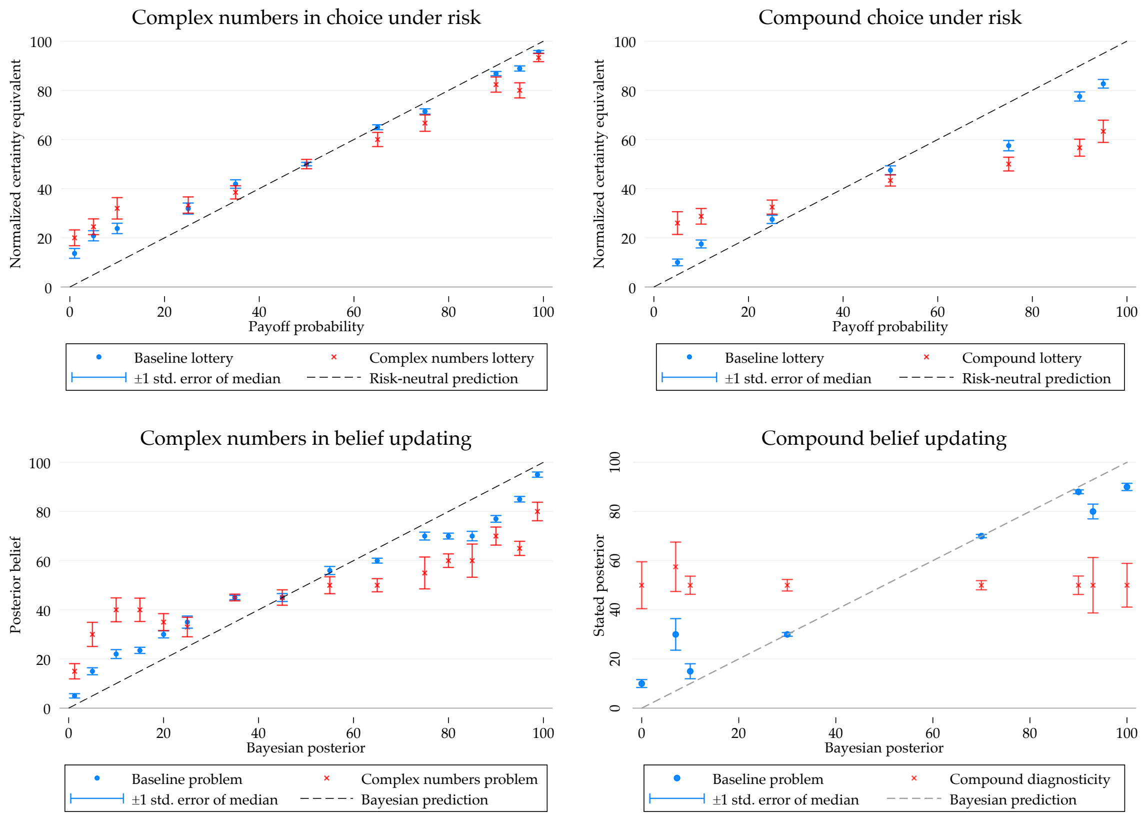


Figure 5: Complexity and decisions. Panel A shows median normalized certainty equivalents separately for baseline and complex numbers lotteries in the *Risk A* experiment ($N = 3,000$). Panel B shows median normalized certainty equivalents separately for baseline and compound lotteries in the *Risk B* experiment ($N = 1,958$). Panel C shows median posterior beliefs separately for baseline and complex numbers updating problems in the *Beliefs A* experiment ($N = 3,000$). Panel D shows median posterior beliefs separately for baseline and compound updating problems in the *Beliefs B* experiment ($N = 2,056$). Whiskers show standard error bars. The beliefs figures show bins with more than 30 observations.

metric biases, such as "base rate neglect parameters" or a "probability weighting sensitivity parameter". Instead, our results suggest that the complexity of the decision environment partly determines the level of cognitive noise, which, in turn, drives the magnitude of errors.

7 Estimating the Central Tendency Effect

The framework laid out in Section 2 asserts that the compression patterns documented in this paper reflect a regression of average behavior to an "intermediate" d, which could either reflect a fixed default (prior) or the mean random choice. Either interpretation is reminiscent of well-established "central tendency effects" in psychological research on judgment and decision-making. Here, we contribute to this discussion by directly estimating the central tendency effect (d), regardless of whether it reflects a fixed prior or the mean random choice. We do not have a general theory of what determines people's priors, though some research in cognitive psychology suggests that the prior may reflect a decision that makes sense on average (e.g., Petzschner et al., 2015; Xiang et al., 2021).[15]

Recall that the average decision in our framework can be expressed as a convex combination of the expected-utility-maximizing decision and d, with the relative weight λ being a function of the magnitude of (unobserved) cognitive noise. We proceed by heuristically approximating $\lambda = max\{0; (1 - \gamma p_{CU})\}$, where γ is a nuisance parameter to be estimated. We can then estimate the decision rule in (2) as:

$$a^o = \underbrace{max\{1 - \gamma p_{CU}; 0\}}_{\lambda} a^*(p) + \underbrace{min\{\gamma p_{CU}; 1\}}_{1-\lambda} d + \epsilon, \tag{4}$$

where p_{CU} is observed, γ and d are to be estimated and ϵ is a disturbance term.[16] The utility-maximizing decision a^* is assumed to be the Bayesian posterior in belief updating. For choice under risk, we assume that the utility-maximizing decision reflects CRRA utility, with utility curvature to be estimated.[17]

We estimate this equation at the population level using standard non-linear least squares techniques. This means that we leverage individual-level (in fact, decision-level) variation in CU but estimate a single average d for the population. For benchmarking purposes, we also estimate a "restricted model" that excludes cognitive noise, i.e., setting $p_{CU} = 0$.

Table 5 reports the model estimates for both our A experiments and the earlier B experiments. There are three main takeaways. First, we consistently estimate an "intermediate" mean of the prior distribution. The estimated cognitive default is very close to 0.5 in the beliefs experiments and somewhat lower at around 0.4 in choice under risk. The estimates of the default decision jive well both with the visual impressions from Figure 3 and with the large body of work on central tendency or compromise effects in psychology and economics.

The second main takeaway from the model estimations is that allowing for a role of

[15]Some research suggests that people's priors may be influenced by a 1/N logic, where N is the number of states (Zhang and Maloney, 2012). To test this idea, we ran additional experiments in which we implemented a partition manipulation: in the belief updating and choice under risk experiments, we increased the number of states from two to ten without changing the normatively relevant features of the problem. Under the assumptions that (i) the model parameter d reflects a fixed prior and (ii) that it is partly influenced by a 1/N logic, such a treatment should decrease observed decisions, and more so for cognitively uncertain people. Appendix F reports the results of these experiments, which are mixed.

[16]Note that in this approach, λ (and hence unobserved cognitive noise) varies at the choice level, but the nuisance parameter γ is fixed at the population level.

[17]The estimating equation with CRRA utility curvature parameter α is given by $a^o = max\{1 - \gamma p_{CU}; 0\} p^{1/\alpha} + min\{\gamma p_{CU}; 1\} d + \epsilon$.

Table 5: Estimates of central tendency effect across experiments

	Risk A		*Beliefs A*		*Risk B*		*Beliefs B*	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Restr.	CU	Restr.	CU	Restr.	CU	Restr.	CU
$\hat{d}$	N/A	0.43	N/A	0.52	N/A	0.40	N/A	0.52
AIC	18958	18477	211	-936	7996	7707	211	-935

Notes. Estimates of different versions of (4). Columns (1), (3), (5) and (7): set $\gamma = 1$ and $p_{CU} = 0$. All estimated standard errors (computed based on clustering at the subject level) are smaller than 0.02.

cognitive noise increases model fit substantially relative to the restricted model that does not include cognitive uncertainty. This can be inferred from the lower values of Akaike's Information Criterion.

8 Discussion

This paper has argued that measuring cognitive uncertainty in a simple, fast and costless manner allows experimental and survey researchers to predict behavior and biases and to shed light on the decision modes that underlie commonalities in errors across different domains. Instead of recapitulating the paper's results, we here discuss extensions, limitations and directions for future research.

Extension: S-shaped response functions. While our main empirical analyses focus on the observation that people's beliefs and choices are *compressed* towards some intermediate value, it is well-known in the literature that decisions are often non-linear (inverse S-shaped) in objective probabilities (see Figure 1). As we discuss in detail in Appendix D, our account of cognitive uncertainty also sheds light on this regularity. The reason is that cognitive uncertainty is hump-shaped in objective probabilities. For example, it appears to be easier for people to value a lottery that has a payout probability close to the boundaries. Similarly, people report lower cognitive uncertainty in belief updating problems that have Bayesian posteriors close to the boundaries. The model estimations in Appendix D show that these non-linearities in how cognitive uncertainty depends on objective probabilities can translate into the canonical S-shaped response functions commonly observed in the literature.

Extension: Ambiguity attitudes. While in this paper we focus on how cognitive uncertainty sheds light on the pattern that people treat different objective probabilities to some degree alike, there is also a direct connection to research on ambiguity. The reason is that recent reviews highlight the concept of "ambiguity-insensitivity," which asserts that people are excessively insensitive to changes in the likelihood of ambiguous events (Trautmann and Van De Kuilen, 2015). In the working paper version of this paper, we document that measured cognitive uncertainty also strongly predicts the magnitude of ambiguity insensitivity (Enke and Graeber, 2019). Indeed, we find that cognitively uncertain people often act as though they are ambiguity-seeking when an ambiguous event is very unlikely.

Implications for research linking expectations measures to field behaviors. If stated expectations are systematically distorted due to the types of compression effects that we document in this paper, demographic differences in expectations could just reflect heterogeneity in cognitive noise rather than true beliefs. Moreover, when researchers estimate links between expectations and field behaviors, cognitive noise could attenuate these relationships. In line with this conjecture, Drerup et al. (2017), Giglio et al. (2019) and Yang (2023) find that the relationship between expectations and investment behavior is considerably more pronounced among people with high confidence in their expectations. We conjecture that cognitive uncertainty will be predictive of the strength of the relationship between behaviors and expectations more generally (see also, e.g., Charles et al., 2022; Yang, 2023). Thus, at a minimum, measuring cognitive uncertainty in surveys allows researchers to conduct heterogeneity analyses regarding the predictability of field behaviors.

Limitations. An obvious limitation of our approach is that we do not have a general theory of what the prior / cognitive default / mean random choice is. We here work with the idea that the mean prior reflects the decision they would have taken prior to deliberating about the problem at hand. Yet, casual introspection suggests that other factors might also shape people's initial reactions. For instance, if a choice option is displayed in red font, it might be visually salient and therefore serve as a cognitive anchor from which people's deliberation process adjusts.

Related to this discussion is research on bounded rationality that focuses on the role of misleading intuitions, as they result from salience, focusing, or memory-based cueing effects (e.g., Kahneman, 2011; Bordalo et al., 2013, 2017; Kőszegi and Szeidl, 2013; Enke et al., 2020). While our paper is more concerned with the effects of complexity than with those of strong intuitions, we conjecture that the (unspecified) cognitive default provides a potential link between these two literatures. We speculate that strong intuitions, salient choice options or associations-based recall shape people's initial reaction to a choice problem (the prior / cognitive default), while cognitive uncertainty captures the degree to which people adjust away from these initial reactions. If true, such a perspective would suggest the testable prediction that salience, focusing and memory-based cueing effects are particularly pronounced among people with high cognitive uncertainty.

More closely integrating cognitive noise with attention and memory research is also relevant because prior work has shown that both probability weighting in risky choice and probability estimates are influenced by salience and asymmetric recall (e.g., Stewart et al., 2006; Bordalo et al., 2012, 2021). Similarly, a broad body of work often identifies the opposite of probability weighting when people decide based on experience rather than problem descriptions (Hertwig and Erev, 2009). It is not obvious that our approach of measuring cognitive uncertainty can reconcile these patterns.

A third limitation of our work is that we do not have a theory of which aspects of a decision actually generate cognitive noise and resulting cognitive uncertainty. As we saw above, more complex decisions lead to higher cognitive uncertainty. Prior work has shown that cognitive noise is also a function of time pressure, experience and prior beliefs (Polania et al., 2019; Prat-Carrabin and Woodford, 2021; Frydman and Jin, 2021). Yet, a general theory of what makes a task (not) complex is not available. Other aspects that generate cognitive uncertainty may pertain to the decision-maker: the availability of cognitive resources or the amount of experience. Future research could helpfully shed light on this.

Open questions and potential applications. We conjecture that the measurement of cognitive uncertainty could shed light on behavior in multiple other domains of economic decision-making. Most fundamentally, people likely don't just have cognitive uncertainty in choosing between lotteries or in updating their beliefs, but also in other domains. For instance, in Enke et al. (2022), we study how cognitive uncertainty helps to shed light on "anomalies" in intertemporal choice. Yet, we speculate that there may be many more applications in which a measurement of cognitive uncertainty could shed light on biases and anomalies that have a "compression" flavor. For example, in the widely-studied newsvendor game that is of relevance to researchers in economics, management and operations research, people generally succumb to a pull-to-the-center bias (Schweitzer and Cachon, 2000). Similarly, laboratory experiments on effort choice often find that the elasticity of labor supply with respect to piece rates is very low; we again speculate that this insensitivity / compression effect could be explained by measuring cognitive uncertainty.

Another open question relates to the link between objective cognitive noise and cognitive uncertainty. In the decision contexts that we study in this paper, people's awareness of their own cognitive noise is at least partly accurate. Yet, in other decision domains, people's meta-cognition may be less well-calibrated, as in Enke et al. (2021). This immediately raises the question of when people's cognitive uncertainty is (not) reflective of actual noise.

Finally, another open question concerns the choice implications of cognitive noise. In this paper, we have highlighted the empirical regularity that cognitive uncertainty is associated with an attenuated relationship between decisions and problem parameters. In other contexts, cognitive uncertainty may predict a form of "caution" (Cerreia-Vioglio et al., 2015) or "complexity aversion," according to which people shy away from choice options regarding which they have high cognitive uncertainty. Future research could helpfully shed light on when compression effects or caution dominate.

References

Agranov, Marina and Pietro Ortoleva, "Stochastic choice and preferences for randomization," *Journal of Political Economy*, 2017, *125* (1), 40–68.

_ **and** _ , "Ranges of Preferences and Randomization," *Working Paper*, 2020.

Beauchamp, Jonathan P, Daniel J Benjamin, David I Laibson, and Christopher F Chabris, "Measuring and controlling for the compromise effect when estimating risk preference parameters," *Experimental Economics*, 2019, pp. 1–31.

Benjamin, Daniel J, "Errors in Probabilistic Reasoning and Judgmental Biases," in "Handbook of Behavioral Economics" 2019.

Bernheim, B Douglas and Charles Sprenger, "Direct tests of cumulative prospect theory," *Available at SSRN 3350196*, 2019.

Bhatia, Sudeep, "Sequential sampling and paradoxes of risky choice," *Psychonomic Bulletin & Review*, 2014, *21* (5), 1095–1111.

_ **and Graham Loomes**, "Noisy preferences in risky choice: A cautionary note.," *Psychological review*, 2017, *124* (5), 678.

Blavatskyy, Pavlo R, "Stochastic expected utility theory," *Journal of Risk and Uncertainty*, 2007, *34* (3), 259–286.

Bordalo, Pedro, John J Conlon, Nicola Gennaioli, Spencer Yongwook Kwon, and Andrei Shleifer, "Memory and probability," Technical Report, National Bureau of Economic Research 2021.

_ **, Nicola Gennaioli, and Andrei Shleifer**, "Salience theory of choice under risk," *Quarterly Journal of Economics*, 2012, *127* (3), 1243–1285.

_ **,** _ **, and** _ , "Salience and Consumer Choice," *Journal of Political Economy*, 2013, *121*, 803–843.

_ **,** _ **, and** _ , "Memory, Attention, and Choice," *Quarterly Journal of Economics*, 2017.

Bruhin, Adrian, Helga Fehr-Duda, and Thomas Epper, "Risk and rationality: Uncovering heterogeneity in probability distortion," *Econometrica*, 2010, *78* (4), 1375–1412.

Butler, David J and Graham C Loomes, “Imprecision as an account of the preference reversal phenomenon,” *American Economic Review*, 2007, *97* (1), 277–297.

Cerreia-Vioglio, Simone, David Dillenberger, and Pietro Ortoleva, “Cautious expected utility and the certainty effect,” *Econometrica*, 2015, *83* (2), 693–728.

Charles, Constantin, Cary Frydman, and Mete Kilic, “Insensitive Investors,” *Available at SSRN 3971082*, 2022.

Cubitt, Robin P, Daniel Navarro-Martinez, and Chris Starmer, “On preference imprecision,” *Journal of Risk and Uncertainty*, 2015, *50* (1), 1–34.

Danz, David, Lise Vesterlund, and Alistair J Wilson, “Belief elicitation: Limiting truth telling with information on incentives,” Technical Report, National Bureau of Economic Research 2020.

Drerup, Tilman, Benjamin Enke, and Hans-Martin Von Gaudecker, “The precision of subjective data and the explanatory power of economic models,” *Journal of Econometrics*, 2017, *200* (2), 378–389.

Enke, Benjamin and Thomas Graeber, “Cognitive Uncertainty,” Working Paper 26518, National Bureau of Economic Research November 2019.

_, Frederik Schwerter, and Florian Zimmermann, “Associative Memory and Belief Formation,” *Working Paper*, 2020.

_, Thomas Graeber, and Ryan Oprea, “Confidence, Self-Selection and Bias in the Aggregate,” *Working Paper*, 2021.

_, _, and _, “Complexity and Time,” *Working Paper*, 2022.

Erev, Ido, Eyal Ert, Ori Plonsky, Doron Cohen, and Oded Cohen, “From anomalies to forecasts: Toward a descriptive model of decisions under risk, under ambiguity, and from experience.,” *Psychological review*, 2017, *124* (4), 369.

_, Thomas S Wallsten, and David V Budescu, “Simultaneous over-and underconfidence: The role of error in judgment processes.,” *Psychological review*, 1994, *101* (3), 519.

Falk, Armin, Anke Becker, Thomas Dohmen, David Huffman, and Uwe Sunde, “The Preference Survey Module: A Validated Instrument for Measuring Risk, Time, and Social Preferences,” *Working Paper*, 2015.

Fehr-Duda, Helga and Thomas Epper, "Probability and risk: Foundations and economic implications of probability-dependent risk preferences," *Annu. Rev. Econ.*, 2012, *4* (1), 567–593.

Fennell, John and Roland Baddeley, "Uncertainty plus prior equals rational bias: An intuitive Bayesian probability weighting function.," *Psychological Review*, 2012, *119* (4), 878.

Fischhoff, Baruch and Wändi Bruine De Bruin, "Fifty–fifty= 50%?," *Journal of Behavioral Decision Making*, 1999, *12* (2), 149–163.

Frydman, Cary and Lawrence Jin, "Efficient Coding and Risky Choice," *Quarterly Journal of Economics*, 2021.

Gabaix, Xavier, "Behavioral inattention," in Douglas Bernheim, Stefano DellaVigna, and David Laibson, eds., *Handbook of Behavioral Economics-Foundations and Applications 2*, Elsevier, 2019, p. 261.

_ **and David Laibson**, "Myopia and discounting," Technical Report, National bureau of economic research 2017.

Giglio, Stefano, Matteo Maggiori, Johannes Stroebel, and Stephen Utkus, "Five facts about beliefs and portfolios," Technical Report, National Bureau of Economic Research 2019.

Gillen, Ben, Erik Snowberg, and Leeat Yariv, "Experimenting with Measurement Error: Techniques with Applications to the Caltech Cohort Study," *Journal of Political Economy*, 2019.

Grether, David M., "Bayes Rule as a Descriptive Model: The Representativeness Heuristic," *Quarterly Journal of Economics*, 1980, *95*, 537–557.

Harbaugh, William T, Kate Krause, and Lise Vesterlund, "The fourfold pattern of risk attitudes in choice and pricing tasks," *The Economic Journal*, 2010, *120* (545), 595–611.

Healy, Paul J, "Explaining the BDM – Or any random binary choice elicitation mechanism – To Subjects," Technical Report, mimeo 2018.

Heng, Joseph A, Michael Woodford, and Rafael Polania, "Efficient sampling and noisy decisions," *Elife*, 2020, *9*, e54962.

Hertwig, Ralph and Ido Erev, "The description–experience gap in risky choice," *Trends in cognitive sciences*, 2009, *13* (12), 517–523.

Hollingworth, Harry Levi, "The central tendency of judgment," *The Journal of Philosophy, Psychology and Scientific Methods*, 1910, *7* (17), 461–469.

Honig, Maija, Wei Ji Ma, and Daryl Fougnie, "Humans incorporate trial-to-trial working memory uncertainty into rewarded decisions," *Proceedings of the National Academy of Sciences*, 2020, *117* (15), 8391–8397.

Hossain, Tanjim and Ryo Okui, "The binarized scoring rule," *Review of Economic Studies*, 2013, *80* (3), 984–1001.

Huck, Steffen and Georg Weizsäcker, "Risk, complexity, and deviations from expected-value maximization: Results of a lottery choice experiment," *Journal of Economic Psychology*, 1999, *20* (6), 699–715.

Kahneman, Daniel, *Thinking, Fast and Slow*, Macmillan, 2011.

Khaw, Mel Win, Ziang Li, and Michael Woodford, "Cognitive imprecision and small-stakes risk aversion," *The review of economic studies*, 2021, *88* (4), 1979–2013.

Kőszegi, Botond and Adam Szeidl, "A model of focusing in economic choice," *Quarterly Journal of Economics*, 2013, *128* (1), 53–104.

Marchiori, Davide, Sibilla Di Guida, and Ido Erev, "Noisy retrieval models of over-and undersensitivity to rare events.," *Decision*, 2015, *2* (2), 82.

Martino, Benedetto De, Sebastian Bobadilla-Suarez, Takao Nouguchi, Tali Sharot, and Bradley C Love, "Social information is integrated into value and confidence judgments according to its reliability," *Journal of Neuroscience*, 2017, *37* (25), 6066–6074.

_ , Stephen M Fleming, Neil Garrett, and Raymond J Dolan, "Confidence in value-based choice," *Nature neuroscience*, 2013, *16* (1), 105–110.

Moore, Don A and Paul J Healy, "The trouble with overconfidence.," *Psychological review*, 2008, *115* (2), 502.

Mukherjee, Kanchan, "A dual system model of preferences under risk.," *Psychological review*, 2010, *117* (1), 243.

O'Donoghue, Ted and Jason Somerville, "Modeling risk aversion in economics," *Journal of Economic Perspectives*, 2018, *32* (2), 91–114.

Oprea, Ryan, "Simplicity equivalents," Technical Report, Working Paper 2022.

Petzschner, Frederike H, Stefan Glasauer, and Klaas E Stephan, "A Bayesian perspective on magnitude estimation," *Trends in cognitive sciences*, 2015, *19* (5), 285–293.

Polania, Rafael, Michael Woodford, and Christian C Ruff, "Efficient coding of subjective value," *Nature neuroscience*, 2019, *22* (1), 134–142.

Prat-Carrabin, Arthur and Michael Woodford, "Bias and variance of the Bayesian-mean decoder," *Advances in Neural Information Processing Systems*, 2021, *34*, 23793–23805.

Schweitzer, Maurice E and Gérard P Cachon, "Decision bias in the newsvendor problem with a known demand distribution: Experimental evidence," *Management Science*, 2000, *46* (3), 404–420.

Simonson, Itamar and Amos Tversky, "Choice in context: Tradeoff contrast and extremeness aversion," *Journal of marketing research*, 1992, *29* (3), 281–295.

Stewart, Neil, Nick Chater, and Gordon DA Brown, "Decision by sampling," *Cognitive psychology*, 2006, *53* (1), 1–26.

Trautmann, Stefan T and Gijs Van De Kuilen, "Ambiguity attitudes," *The Wiley Blackwell handbook of judgment and decision making*, 2015, *1*, 89–116.

Tversky, Amos and Daniel Kahneman, "Judgment under uncertainty: Heuristics and biases," *science*, 1974, *185* (4157), 1124–1131.

_ and _ , "Advances in prospect theory: Cumulative representation of uncertainty," *Journal of Risk and uncertainty*, 1992, *5* (4), 297–323.

Viscusi, W Kip, "A Bayesian perspective on biases in risk perception," *Economics Letters*, 1985, *17* (1-2), 59–62.

_ , "Prospective reference theory: Toward an explanation of the paradoxes," *Journal of risk and uncertainty*, 1989, *2* (3), 235–263.

Wakker, Peter P, *Prospect theory: For risk and ambiguity*, Cambridge university press, 2010.

Woodford, Michael, "Modeling Imprecision in Perception, Valuation and Choice," *Working Paper*, 2019.

_ , "Modeling imprecision in perception, valuation, and choice," *Annual Review of Economics*, 2020, *12*, 579–601.

Xiang, Yang, Thomas Graeber, Benjamin Enke, and Samuel Gershman, "Confidence and Central Tendency in Perceptual Judgment," *Attention, Perception and Psychophysics*, 2021.

Yang, Jeffrey, "On the Decision-Relevance of Subjective Beliefs," *Working Paper*, 2023.

Zhang, Hang and Laurence T Maloney, "Ubiquitous log odds: a common representation of probability and frequency distortion in perception, action, and cognition," *Frontiers in neuroscience*, 2012, *6*, 1.

_ **, Xiangjuan Ren, and Laurence T Maloney**, "The bounded rationality of probability distortion," *BioRxiv*, 2020, p. 662429.

ONLINE APPENDIX

A Theoretical Framework

A.1 Baseline Model

Below we discuss the main behavioral predictions of a Bayesian cognitive noise model as outlined in Section 2. Suppose the DM does not know their ex-ante utility-maximizing action, a^*, but has access to a mental simulation, S, which is an unbiased cognitive signal of a^*,

$$S \sim \frac{1}{N} Bin(N, a^*), \tag{5}$$

such that $0 \leq S \leq 1$. The parameter N controls the precision of the mental simulation.

The DM holds a prior about his subjective utility-maximizing action, a^*. We assume that this prior can be represented by a Beta distribution, $a^* \sim Beta(nd\ ,\ n(1-d))$. Here, d is the prior mean and carries the interpretation of a "cognitive default" action that the DM would take before deliberating about the problem. The parameter n, on the other hand, reflects the DM's confidence in (or precision of) his prior.[18] Given the fact she has a prior, the cognitive signal from the DM's perspective is seen as:

$$S \sim \frac{1}{N} Bin(N, a^*). \tag{6}$$

The subjective likelihood of choosing the true utility-maximizing action based on a randomly drawn cognitive signal $\{S = s\}$ can then be represented by a binomial distribution:

$$\mathscr{L}(a^*|S = s) = P(S = s|a^*, N) = \binom{N}{sN} (a^*)^{sN} (1 - a^*)^{(1-s)N}. \tag{7}$$

A Bayesian DM accounts for the noisiness of his mental simulation by implicitly forming a posterior assessment of the utility-maximizing action. Given a Beta-distributed prior and a Binomial signal, this posterior belief, $a^*|S = s$, is also Beta-distributed.[19] We assume the

[18]Note that $n = a + b$ is a re-parameterization of the shape parameters a and b of the Beta distribution. n is inversely related to the variance of the prior, $\sigma_A^2 = \frac{d \cdot (1-d)}{1+n}$.

[19]Specifically, $a^*|S = s \sim\ Beta(sN + nd\ ,\ N(1-s) + n(1-d))$.

DM's decision is given by his posterior mean:[20]

$$a^o = E[a^*|S = s] = \lambda s + (1-\lambda)d \quad \text{with} \quad \lambda = N/(n+N). \tag{8}$$

Crucially, a more precise mental simulation (higher N) has a direct effect on the weighting factor λ which implies a lower weight on the cognitive default action. In the following subsection, we will thus focus on deriving behavioral predictions for changes in λ. In subsection A.3 we characterize cognitive uncertainty in the context of this model.

For the purposes of the belief updating experiments it will be helpful to define the following terms:

$$b := \text{ The prior / base rate} \tag{9}$$

$$h := \text{ The signal diagnosticity} \tag{10}$$

$$d := \text{ The cognitive default decision} \tag{11}$$

$$n := \text{ The number of balls in the sample} \tag{12}$$

$$k := \text{ The number of red balls in the sample} \tag{13}$$

These quantities allows us to derive / define the following quantities:

$$a^* := \frac{h^k(1-h)^{n-k}b}{h^k(1-h)^{n-k}b + (1-h)^k h^{n-k}(1-b))} \qquad \text{(Bayesian Posterior)}$$

$$a := \lambda a^* + (1-\lambda)d \qquad \text{The mean observed action}$$

$$o := \frac{b}{1-b} \qquad \text{The prior odds}$$

$$LR := \frac{h^k(1-h)^{n-k}}{(1-h)^k h^{n-k}} = \left(\frac{h}{1-h}\right)^{2k-n} \qquad \text{The likelihood ratio}$$

$$lo := \frac{a}{1-a} \qquad \text{Log[Subjective posterior odds]}$$

[20]We focus on the mean for tractability. This is precisely the optimal response in our belief formation experiments, given the quadratic loss function implied by our scoring rule. However, in both our risk and beliefs experiments, even people's subjectively expected reward from a given response might in principle also depend on risk preferences. Put differently, subjects may be risk-averse vis-à-vis their subjective distribution about the optimal action. If one makes this assumption, the optimal response is ambiguous in both kinds of experiments. Whichever view one takes, in the present theoretical setup, the approximation error from assuming the subject playing the subjective mean is likely small. For instance, the mean of a Beta(a,b) variable is $a/(a+b)$, whereas the mode is $(a-1)/(a+b-2)$, and the median lies between the two.

A.2 Proofs of Predictions in Main Text

We restate the predictions from the main text more formally here and provide proofs.

Prediction 1 (Cognitive noise and compression effects)**.**

(a) *In risky choice, cognitive noise is correlated with probability weighting. Specifically, the mean error,* $e := \mathbb{E}[a^o - p]$*, when faced with the same situation satisfies:* $\partial e / \partial \lambda < 0$ *for* $p < u(d)$ *and* $\partial e / \partial \lambda > 0$ *for* $p > u(d)$*.*

(b) *i. In stated beliefs, cognitive noise is correlated with overestimation of small and underestimation of large probabilities. Specifically, the mean error,* $e := \mathbb{E}[a^o - p]$*, when faced with the same situation satisfies:* $\partial e / \partial \lambda < 0$ *for* $p < d$ *and* $\partial e / \partial \lambda > 0$ *for* $p > d$*.*

ii. In Grether decompositions, when taking the default position to be in the interior $d \in (0, 1)$*, cognitive noise is correlated with base rate insensitivity and conservatism (likelihood ratio insensitivity).*

Proof.

(a) We consider the expression for e, that is, $\mathbb{E}[a^o - p]$ and compute the derivative:

$$\frac{\partial e}{\partial \lambda} = a^* - d \tag{14}$$

hence, we see that $\frac{\partial e}{\partial \lambda} > 0$ when:

$$a^* > d \tag{15}$$

$$p > u(d) \tag{16}$$

The result immediately follows.

(b) *i.* This follows given the result above and noting that $a^* = p$, that is, the utility maximizing choice is the Bayesian posterior, by *mutatis mutandis*.

ii. In a Grether regression framework, our prediction concerns how the derivative of the log posterior odds with respect to the log prior odds depends on λ. Base rate insensitivity that increases in cognitive noise would mean that:

$$\frac{\partial^2 lo}{\partial \lambda \partial \ln|o|} \geq 0 \tag{17}$$

and we begin by deriving this inequality. Since b is not a function of λ, the desired derivative may be expanded as:

$$\frac{\partial^2 lo}{\partial \lambda \partial \ln|o|} = \frac{\partial^2 lo}{\partial \lambda \partial b} \frac{db}{d \ln|o|} \tag{18}$$

Noting that:

$$b = \frac{e^{\ln|o|}}{1 + e^{\ln|o|}} \tag{19}$$

so that we find:

$$\frac{\partial b}{\partial \ln|o|} = \frac{e^{\ln|o|}}{(1 + e^{\ln|o|})^2} \tag{20}$$

$$= b(1-b) \geq 0 \tag{21}$$

Accordingly, our claim will be proven if:

$$\frac{\partial^2 lo}{\partial \lambda \partial b} \geq 0 \tag{22}$$

For simplicity, we define the following quantities:

$$e_1 = b^2(1-h)^{2n}h^{4k}(\lambda^2(1-d)^2 + d(1-d)) \tag{23}$$

$$e_2 = 2((1-h)h)^{2k+n}b(1-b)(1-\lambda^2)d(1-d) \tag{24}$$

$$e_3 = (1-h)^{4k}h^{2n}(1-b)^2 d(1-d(1-\lambda^2)) \tag{25}$$

and it may be seen that for $0 \leq k \leq n$ and $b, h, \lambda, d \in (0,1)$ that the quantities above are positive.

We then proceed to directly compute the value of this mixed partial and after combining and canceling out terms find it to be:

$$\frac{\partial^2 lo}{\partial \lambda \partial b} = \frac{((1-h)h)^{2k+n}(e_1 + e_2 + e_3)}{x^2 y^2} \tag{26}$$

where we have

$$x = (1-h)^{2k}h^n(1+d(1-\lambda)(1-b)-(1-d)(1-\lambda)(1-h)^n h^{2k} b) \quad (27)$$

$$y = (1-h)^n h^{2k}\lambda b + d(\lambda-1)((1-h)^{2k}h^n(b-1)-(1-h)^n h^{2k} b) \quad (28)$$

The denominator is the product of two squares and is accordingly non-negative. Since the terms $(1-h)h, e_1, e_2, e_3 > 0$ given our assumptions, we have accordingly shown that base rate insensitivity decreases in signal precision, λ. In other words, base rate insensitivity increases in cognitive noise, $(1-\lambda)$.

We now consider likelihood ratio insensitivity. If we define the log of the subject's log posterior odds as lo, then likelihood insensitivity that increases in cognitive noise would mean that:

$$\frac{\partial^2 lo}{\partial\lambda\partial\ln|LR|} \geq 0 \quad (29)$$

and we again begin by deriving this inequality. Since h is not a function of λ, the desired derivative may be expanded as:

$$\frac{\partial^2 lo}{\partial\lambda\partial\ln|LR|} = \frac{\partial^2 lo}{\partial\lambda\partial h}\frac{dh}{d\ln|LR|} \quad (30)$$

Now, the sign of $dg/d\ln|LR|$ is the same[21] as that of $d|LR|/dh$ and we see that the latter may computed to be:

$$\frac{d|LR|}{dh} = (2k-n)\left(\frac{h^{2k-n-1}}{(1-h)^{2k-n+1}}\right) \quad (31)$$

hence, its sign depends on that of $2k-n$. Accordingly, our claim will be proven if:

$$\operatorname{sgn}\left(\frac{\partial^2 lo}{\partial\lambda\partial h}\right) = \operatorname{sgn}(2k-n) \quad (32)$$

We may directly compute the value of this mixed partial and after combining and canceling out terms find it to be:

$$\frac{\partial^2 lo}{\partial\lambda\partial h} = \frac{(2k-n)((1-h)h)^{2k+n-1}b(1-b)(e_1+e_2+e_3)}{x^2y^2} \quad (33)$$

[21]Recall, that $\frac{d\ln|LR|}{dh} = \frac{1}{|LR|}\frac{d|LR|}{dh}$.

Once again the denominator is the product of two squares and is accordingly non-negative. Since the terms $(1-h)h, e_1, e_2, e_3 > 0$ given our assumptions, we have accordingly proven our claim.

□

Prediction 2. *The squared difference between the DM's decisions and his utility-maximizing decision decreases in signal precision on average when the signal is more informative than the prior. Stated formally, we have that the mean squared error*

$$\mathbb{E}[(a^o - a^*)^2] \tag{34}$$

satisfies:

$$\partial \mathbb{E}[(a^o - a^*)^2]/\partial N < 0 \tag{35}$$

when $N > n$.

Proof. We recall that for a given task:

$$a^o = \lambda S + (1-\lambda)d$$

where $NS \sim Bin(N, a^*)$ and $\lambda = N/(n+N)$. Accordingly, we may compute:

$$\mathbb{E}[(a^o - a^*)^2] = \frac{d^2n^2 - 2dn^2a^* + a^*(N + n^2a^* - Na^*)}{(n+N)^2} \tag{36}$$

taking the derivative we find that:

$$\frac{\partial \mathbb{E}[(a^o - a^*)^2]}{\partial N} = \frac{(N-n)a^*(a^*-1) - 2n^2(a^*-d)^2}{(n+N)^3} \tag{37}$$

which is negative since $N > n$ and $a^* < 1$ thereby proving the claim. □

A.3 Cognitive Uncertainty and Cognitive Noise

As laid out in Section 2, the DM subjectively perceives his ex-ante utility-maximizing decision as a *distribution* conditional on his noisy signal. This means: while the agent is assumed to choose $a^o = \mathbb{E}[A_{S=s}]$, the underlying perceived posterior distribution of the utility-maximizing decision is Beta-distributed:

$$A_{S=s} \sim \; Beta\left(\underbrace{sN + nd}_{\equiv a},\; \underbrace{N(1-s) + n(1-d)}_{\equiv b}\right) \tag{38}$$

where N is the signal precision. Now, let us restate our definition of cognitive uncertainty,

$$p_{CU} := \mathbb{P}(|A_{S=s} - \mathbb{E}[A_{S=s}]| > \kappa), \tag{39}$$

for fixed constant κ. The objective of this subsection is to establish that increases in signal precision decrease cognitive uncertainty. Below, we develop two sets of results about this relationship. First, Corollary 1 provides a limit argument showing that any desired decrease in cognitive uncertainty can be achieved by an increase in signal precision. Second, to shed light on the case with low signal precision, Theorem 1 shows that cognitive uncertainty decreases with signal precision when using the Gaussian approximation of the Beta distribution.

To begin, we prove:

Proposition 1. *$\forall \kappa > 0, \forall \varepsilon > 0, \exists N^* \in \mathbb{N}$ such that $p_{CU} < \varepsilon$ for $N > N^*$.*

Proof. By Chebyshev's inequality we see that for any positive number, κ:

$$p_{CU} < \frac{\text{Var}(A_{S=s})}{\kappa^2} \tag{40}$$

and, since $A_{S=s} \sim Beta(Ns + nd, N(1-s) + n(1-d))$ its variance is found to be:

$$\text{Var}(A_{S=s}) = \frac{(Ns + nd)(N(1-s) + n(1-d)),}{(n+N)^2(n+N+1)} = O(N^{-1}) \tag{41}$$

Accordingly, we find

$$\lim_{N\to\infty} p_{CU} = 0, \tag{42}$$

which in turn yields the proposition. □

This proposition yields the following corollary:

Corollary 1. *Holding the signal value constant $\{S = s\}$ and given a base level of signal precision, N, there exists a constant Δn such that a desired decrease in cognitive uncertainty may be accomplished by increasing the signal precision by more than Δn.*

Formally, given a base signal precision, N, and a desired decrease in cognitive uncertainty, $\delta \in (0, p_{CU})$, there exists a quantity, $\Delta n \in \mathbb{N}$, such that:

$$N\prime > N + \Delta n \to p_{CU} - p_{CU}\prime > \delta \tag{43}$$

with $N\prime$ and $p_{CU}\prime$ being the new signal precision and cognitive uncertainty respectively.

Proof. Given a signal precision N and cognitive uncertainty, p_{CU}, we may apply the proposition to $\varepsilon = p_{CU} - \delta$. We then find that $\Delta n = N - N$. The result follows. □

In essence, this corollary formally states the intuition that any desired decrease in cognitive uncertainty may be accomplished through an increase in signal precision.

In general, though, we will employ the standard Gaussian approximation,[22] which provides a good approximation when $\alpha = \beta$ even for smaller values of α, β. Under the Gaussian approximation we may illustrate our claim concerning the decrease of p_{CU} with respect to signal precision as follows:

Theorem 1. *Holding the signal $\{S = s\}$ constant, an increase in signal precision causes a decrease in cognitive uncertainty in the Gaussian approximation:*

$$\frac{\Delta p_{CU}}{\Delta N} < 0 \tag{44}$$

Proof. This trivially follows from the definition of p_{CU}, the fact that κ is constant and the fact that the variance decreases. □

A.4 Noisy Coding of Probabilities

We show that similar predictions to those presented in Section 2 can be generated by an alternative model of noisy cognition proposed by Khaw et al. (2021). Khaw et al. (2021) examine the effect of cognitive noise in choice under risk. Their model differs from ours in three main ways. First, instead of modeling noisy cognition of the rational action, they propose noisy coding of individual problem parameters. We here focus on noisy coding of payoff probabilities, as considered in Appendix G of Khaw et al. (2021). Second, they assume that individuals subjectively represent probabilities in log-odds form of in line with

[22]This is a commonly used approximation that follows from the fact that $X/(X+Y)$ has a Beta distribution if X, Y are Gamma(λ,1) random variables; that the Gamma distribution is asymptotically normal and an application of the Delta method.

evidence from cognitive psychology (e.g., Zhang and Maloney, 2012), which maps probabilities from the $[0,1]$ interval onto an unbounded log-odds space. Third, they work with normal distributions, unlike our beta-binomial setup. We here apply their model to our setup and develop similar predictions.

The decision maker is asked to indicate his certainty equivalent for a lottery that pays \$1 with probability p and nothing otherwise, but only conceives p with noise. More precisely, we assume that he believes the log-odds $z(p) = \log \frac{p}{1-p}$ follow a prior distribution:

$$z(p) \sim N(\mu_{z(p)}, \sigma_p^2). \tag{45}$$

He then receives an unbiased signal for the log-odds following:

$$S \sim N(z(p), \nu_p^2) \tag{46}$$

After Bayesian updating, the agent's posterior for log-odds will be:

$$\begin{gathered} z(p) \mid S \sim N(m(S), \bar{\sigma}^2) \\ \text{where} \quad m(S) := \mu_{z(p)} + \beta_p \left(S - \mu_{z(p)}\right), \quad \beta_p := \frac{\sigma_p^2}{\sigma_p^2 + \nu_p^2} \quad \text{and} \quad \bar{\sigma}^2 := \frac{\sigma_p^2 \nu_p^2}{\sigma_p^2 + \nu_p^2}. \end{gathered} \tag{47}$$

To model the utility-maximizing decision, we follow Khaw et al. (2021) in assuming that utility is linear, at least locally, so that the rational action for a known p becomes $a^*(p) = p$. Conditional on a draw of the signal S, the decision maker's optimal certainty equivalent is the expected value of p given S:

$$a^o(S) = \mathrm{E}[p \mid S] = \mathrm{E}\left[\frac{e^{z(p)}}{1 + e^{z(p)}} \,\middle|\, S \right], \tag{48}$$

where the integrand is simply the inverse of the log-odds function. An inspection of the posterior distribution of $z(p)$ shows that $a^o(S)$ can be rewritten as the unconditional expectation:

$$a^o(S) = \mathrm{E}\left[\frac{e^{m(S)+\bar{\sigma}\epsilon}}{1 + e^{m(S)+\bar{\sigma}\epsilon}} \right] \quad \text{with } \epsilon \sim N(0,1). \tag{49}$$

Note that $a^o(S)$ is a random variable as it depends on realizations of the signal S.

The value at which the DM is equally likely to accept or refuse the lottery corresponds to the median of the certainty equivalent. As $a^o(S)$ is an increasing function of S, this median will be given by its value at the median value of S, which is the prior mean $\mu_{z(p)}$. Writing out the median certainty equivalent $w(p)$,

$$w(p) := \text{Median}[a^o(S)] = a^o(z(p)) = \mathrm{E}[p \mid S = z(p)] = \mathrm{E}\left[\frac{e^{m(z(p))+\bar{\sigma}\epsilon}}{1+e^{m(z(p))+\bar{\sigma}\epsilon}}\right] \tag{50}$$

Next, we state and prove a number of predictions that are close analogues of the predictions about risky choice problems presented in our main model. Our main predictions in Section 2 apply globally for any level of cognitive noise. Because a similar level of generality is intractable here, we proceed by examining the effect of moving from a situation without cognitive noise to one with cognitive nose.

Prediction 1' (Cognitive noise about probabilities and compression effects).
In risky choice, the presence of cognitive noise is correlated with probability weighting. Specifically, with cognitive noise on probabilities, the median error $e := \text{Median}\big[a^o(S) - p\big]$ is positive for $p < p^*$ and negative for $p^* < p$, for some p^*.

Proof. As in Appendix G of Khaw et al. (2021), one can show that $w(p)$ is continuous, strictly increasing, that $w(p) \to 0$ as $p \to 0$, that $w(p) \to 1$ as $p \to 1$ and that $w'(p) \to +\infty$ as $p \to 0$ or $p \to 1$. It thus exhibits the typical "inverse-S" shape and intersects the 45° line at a unique fixed point $w(p^*) = p^*$ over $(0,1)$. This, in turn, shows that the median error $e = \text{Median}[a^o(S) - p] = w(p) - p$ is positive below p^* and negative above p^*: agents with cognitive noise exhibit probability weighting, while agents without cognitive noise do not. □

Prediction 2' (Cognitive noise and error variance).
The presence of cognitive noise is associated with larger squared differences between the DM's decisions and his utility-maximizing decision than the absence of cognitive noise.

Proof. While $a^o = a^*(p)$ holds absent cognitive noise, in the presence of cognitive noise decisions exhibit a strictly positive mean square error which notably depends on the posterior variance $\bar{\sigma}^2$:

$$\mathrm{E}\big[(a^o(S) - a^*(p))^2\big] = \mathrm{E}\left[\left(\frac{e^{m(S)+\bar{\sigma}\epsilon}}{1+e^{m(S)+\bar{\sigma}\epsilon}} - \frac{e^{z(p)}}{1+e^{z(p)}}\right)^2\right] > 0 \tag{51}$$

□

Although in a model with noisy perception of probabilities there is, strictly speaking, no cognitive default d in the action space, the rational action corresponding to the mean of the prior, which we call probabilistic default and write $d_p := a^*(\mu_{z(p)}) = \frac{e^{\mu_{z(p)}}}{1+e^{\mu_{z(p)}}}$, plays an analogous role.

B Additional Figures

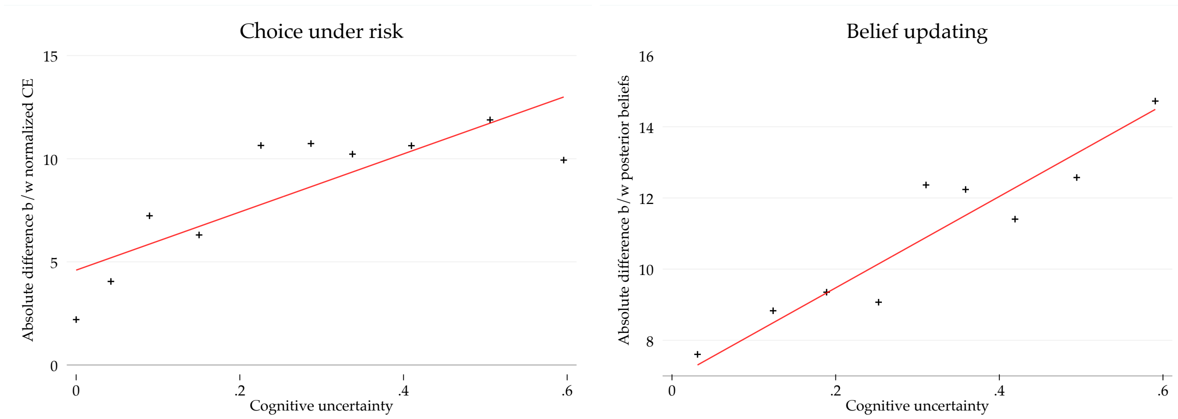


Figure 6: Link between cognitive uncertainty and across-task variability in decisions in *Risk A* (left panel, $N = 1,000$) and *Beliefs A* (right panel, $N = 1,000$). The y-axis captures the average absolute difference between the decisions that a subject took across two implementations of the exact same problem configuration. Cognitive uncertainty is winsorized at the 90th percentile for ease of readability.

C Additional Tables

Table 6: Cognitive uncertainty in belief updating as a function of sample size

	Dependent variable: Cognitive uncertainty			
	(1)	(2)	(3)	(4)
Sample size (Total number of drawn balls)	0.013***	0.014***	0.014***	0.015***
	(0.00)	(0.00)	(0.00)	(0.00)
Absolute difference between number of red and blue balls	-0.017***	-0.016**		
	(0.01)	(0.01)		
Distance b/w Bayesian posterior and 50			-0.0032***	-0.0032***
			(0.00)	(0.00)
Constant	0.34***	0.34***	0.41***	0.42***
	(0.01)	(0.03)	(0.01)	(0.03)
Demographic controls	No	Yes	No	Yes
Observations	4602	4602	4602	4602
R^2	0.00	0.03	0.04	0.06

Notes. Demographic controls include age, gender, college education and performance on the Raven matrices test. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 7: ORIV regressions for *Risk* experiment

	Dependent variable: Normalized certainty equivalent			
	OLS		IV	
	(1)	(2)	(3)	(4)
Payout probability	0.74***	0.74***	0.72***	0.73***
	(0.04)	(0.04)	(0.07)	(0.07)
Payout probability × Cognitive uncertainty	-0.87***	-0.87***	-0.80***	-0.84***
	(0.11)	(0.11)	(0.28)	(0.29)
Cognitive Uncertainty	35.4***	33.0***	31.8**	31.8**
	(7.83)	(7.72)	(14.07)	(14.05)
Constant	18.4***	27.7***	19.4***	28.1***
	(2.72)	(4.90)	(4.20)	(6.07)
Demographic controls	No	Yes	No	Yes
Observations	2040	2040	2040	2040

Notes. Demographic controls include age, gender, college education and performance on the Raven matrices test. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. In the IV estimates, the interaction between the payout probability and cognitive uncertainty is instrumented for by the interaction of the payout probability and cognitive uncertainty in a repeated elicitation of the same lottery valuation problem. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 8: ORIV regressions for *Beliefs* experiment

	Dependent variable: Posterior belief			
	OLS		IV	
	(1)	(2)	(3)	(4)
Bayesian posterior	0.72***	0.72***	0.74***	0.74***
	(0.03)	(0.03)	(0.03)	(0.03)
Bayesian posterior × Cognitive uncertainty	-0.45***	-0.45***	-0.51***	-0.52***
	(0.07)	(0.07)	(0.11)	(0.10)
Cognitive Uncertainty	12.7***	13.1***	15.7***	16.0***
	(4.21)	(4.20)	(5.24)	(5.18)
Constant	19.3***	19.6***	18.4***	18.9***
	(1.85)	(2.75)	(2.07)	(2.87)
Demographic controls	No	Yes	No	Yes
Observations	3060	3060	3050	3050

Notes. Demographic controls include age, gender, college education and performance on the Raven matrices test. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. In the IV estimates, the interaction between the Bayesian posterior and cognitive uncertainty is instrumented for by the interaction of the Bayesian posterior and cognitive uncertainty in a repeated elicitation of the same belief updating problem. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 9: Cognitive uncertainty and likelihood insensitivity in economic forecasts

	Dependent variable:			
	Posterior belief		Ln [Posterior odds]	
	(1)	(2)	(3)	(4)
Historical probability	0.64***	0.63***		
	(0.04)	(0.04)		
Historical probability × Cognitive uncertainty	-0.49***	-0.47***		
	(0.07)	(0.07)		
Cognitive Uncertainty	9.51**	10.3**	-0.76***	-0.67***
	(3.90)	(4.06)	(0.19)	(0.20)
Ln [Historical odds]			0.55***	0.55***
			(0.04)	(0.04)
Ln [Historical odds] × Cognitive uncertainty			-0.47***	-0.47***
			(0.06)	(0.06)
Constant	17.5***	6.18	-0.076	-0.77***
	(2.57)	(4.15)	(0.12)	(0.27)
Demographic controls	No	Yes	No	Yes
Observations	1000	1000	1000	1000
R^2	0.34	0.35	0.30	0.31

Notes. Demographic controls include age, gender, college education and performance on the Raven matrices test. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 10: Complex numbers manipulation in *Risk A*

	Dependent variable: Normalized certainty equivalent			
	(1)	(2)	(3)	(4)
Payout probability	0.62***	0.62***	0.71***	0.70***
	(0.02)	(0.02)	(0.03)	(0.03)
Payout probability × 1 if *Complex numbers*	-0.23***	-0.23***	-0.16***	-0.16***
	(0.04)	(0.04)	(0.04)	(0.04)
1 if *Complex numbers*	6.83**	6.80**	5.26*	5.44*
	(2.80)	(2.75)	(2.85)	(2.80)
Payout probability × Cognitive uncertainty			-0.39***	-0.39***
			(0.06)	(0.06)
Cognitive Uncertainty			7.68*	6.02
			(4.55)	(4.53)
Constant	24.9***	32.4***	23.4***	32.1***
	(1.90)	(4.49)	(2.47)	(4.62)
Demographic controls	No	Yes	No	Yes
Observations	3000	3000	3000	3000
R^2	0.37	0.38	0.40	0.41

Notes. Demographic controls include age, gender, college education and performance on the Raven matrices test. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 11: Compound lottery manipulation in *Risk B*

	Dependent variable: Normalized certainty equivalent			
	(1)	(2)	(3)	(4)
Probability of payout	0.62***	0.62***	0.67***	0.66***
	(0.02)	(0.02)	(0.02)	(0.02)
Payout probability × 1 if compound lottery	-0.30***	-0.29***	-0.27***	-0.27***
	(0.03)	(0.03)	(0.03)	(0.03)
1 if compound	12.3***	12.3***	11.7***	11.6***
	(1.89)	(1.90)	(1.91)	(1.91)
Probability of payout × Cognitive uncertainty			-0.30***	-0.29***
			(0.06)	(0.06)
Cognitive uncertainty			8.07**	7.53*
			(3.95)	(3.96)
Demographic controls	No	Yes	No	Yes
Observations	1918	1918	1918	1918
R^2	0.44	0.45	0.45	0.46

Notes. Demographic controls include age, gender, college education and performance on the Raven matrices test. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 12: Complex numbers manipulations in *Beliefs A*

	Dependent variable:					
	Posterior belief		Ln [Posterior odds]			
	(1)	(2)	(3)	(4)	(5)	(6)
Bayesian posterior	0.60***	0.70***				
	(0.02)	(0.03)				
Bayesian posterior × 1 if *Complex numbers*	-0.27***	-0.20***				
	(0.04)	(0.04)				
1 if *Complex numbers*	9.57***	7.99***	-0.23***	-0.14*	-0.23***	-0.15*
	(2.23)	(2.34)	(0.08)	(0.08)	(0.07)	(0.08)
Bayesian posterior × Cognitive uncertainty		-0.37***				
		(0.07)				
Cognitive Uncertainty		6.89*		-0.60***		-0.59***
		(3.81)		(0.16)		(0.15)
Ln [Bayesian odds]			0.47***	0.55***		
			(0.03)	(0.03)		
Ln [Bayesian odds] × 1 if *Complex numbers*			-0.21***	-0.13***		
			(0.04)	(0.04)		
Ln [Bayesian odds] × Cognitive uncertainty				-0.32***		
				(0.07)		
Log[Prior Odds]					0.62***	0.73***
					(0.04)	(0.05)
Ln [Prior odds] × 1 if *Complex numbers*					-0.26***	-0.15**
					(0.05)	(0.06)
Log[Likelihood Ratio]					0.31***	0.32***
					(0.03)	(0.04)
Ln [Likelihood ratio] × 1 if *Complex numbers*					-0.14***	-0.12***
					(0.04)	(0.04)
Ln [Prior odds] × Cognitive uncertainty						-0.48***
						(0.09)
Ln [Likelihood ratio] × Cognitive uncertainty						-0.082
						(0.09)
Constant	22.4***	18.2***	0.10*	0.22	0.11**	0.23
	(1.61)	(3.48)	(0.05)	(0.19)	(0.05)	(0.18)
Demographic controls	No	Yes	No	Yes	No	Yes
Observations	3018	3018	3018	3018	3018	3018
R^2	0.37	0.40	0.33	0.36	0.37	0.40

Notes. Demographic controls include age, gender, college education and performance on the Raven matrices test. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 13: Compound diagnosticity manipulation in *Beliefs B*

	Dependent variable:					
	Posterior belief		Ln [Posterior odds]			
	(1)	(2)	(3)	(4)	(5)	(6)
Bayesian posterior	0.72***	0.80***				
	(0.02)	(0.02)				
Bayesian posterior × 1 if *Compound Lottery*	-0.51***	-0.47***				
	(0.03)	(0.03)				
1 if *Compound Lottery*	26.4***	25.4***	0.0051	0.033	0.0058	0.034
	(1.75)	(1.77)	(0.05)	(0.05)	(0.05)	(0.05)
Bayesian posterior × Cognitive uncertainty		-0.28***				
		(0.05)				
Cognitive uncertainty		10.5***		-0.14		-0.14
		(3.05)		(0.09)		(0.09)
Log[Posterior Odds]			0.43***	0.48***		
			(0.02)	(0.02)		
Ln [Bayesian odds] × 1 if *Compound Lottery*			-0.26***	-0.24***		
			(0.02)	(0.02)		
Ln [Bayesian odds] × Cognitive uncertainty				-0.20***		
				(0.04)		
Log [Likelihood ratio]					0.45***	0.50***
					(0.02)	(0.02)
Log [Likelihood ratio] × 1 if *Compound Lottery*					-0.28***	-0.25***
					(0.02)	(0.02)
Log [Likelihood ratio] × Cognitive uncertainty						-0.21***
						(0.04)
Constant	15.0***	16.0***	0.052*	0.25**	0.051*	0.25**
	(0.95)	(2.66)	(0.03)	(0.12)	(0.03)	(0.12)
Demographic controls	No	Yes	No	Yes	No	Yes
Observations	1947	1947	1890	1890	1890	1890
R^2	0.60	0.61	0.52	0.53	0.53	0.54

Notes. Demographic controls include age, gender, college education and performance on the Raven matrices test. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

D On Inverse S-Shapes

The objective of this section is to shed light on the pronounced non-linearities (inverse S-shapes) in the response patterns established in the empirical literatures that we built on. After all, the neo-additive function that we derived in Section 2 posits that decisions are a *linear* function of objective probabilities. While these linear representations are popular due to their simplicity, they have the drawback that they do not capture the canonical inverse S-shaped response patterns summarized in Figure 1.

In particular, recall that the estimating equation for our model estimates in eq. (4) linear in p (when $a^*(p)$ is linear):

$$a^o = max\{1-\gamma p_{CU};0\}a^*(p)+min\{\gamma p_{CU};1\}d+\epsilon, \tag{52}$$

However, a crucial observation is that, empirically, cognitive uncertainty, p_{CU}, is not constant across problems with varying objective probability, p. The left-hand panels of Figure 7 show the empirical relationship between measured cognitive uncertainty and objective probabilities in our main experiments. In the top left panel, the x-axis shows the objective payout probability of a gamble. In the bottom left panel, the x-axis denotes the Bayesian posterior in belief updating tasks. Across domains, cognitive uncertainty exhibits a pronounced hump shape: our experimental participants tell us that they find it easier to think about lotteries with extreme payout probabilities, or about belief formation tasks that have extreme solutions. Intuitively, such higher cognitive noise at intermediate probabilities may generate the well-known empirical pattern that decisions are less sensitive to variation in objective probabilities over the intermediate probability range.

To investigate whether these non-linearities could generate an inverse S-shaped decision function, we return to our model estimations. In this regard, it should be pointed out that the optimal decision in (52) only follows from the assumptions stated in Section 2 and Appendix A if the magnitude of cognitive noise does not depend on p. When cognitive noise depends on p, the optimal decision need not be linear under the distributional assumptions that we use. However, as Khaw et al. (2021) show, inverse-S shapes are readily accommodated by cognitive noise models under alternative distributional assumptions. Thus, we heuristically use the linear equation (52) in an ad hoc fashion also when we study the implications of a dependence of CU on p.

To reduce the role of attenuating measurement error in the cognitive uncertainty measurement, we re-estimate equation (4) by replacing each participant's stated CU for a given decision problem with the average CU for a given objective probability p. For example, in

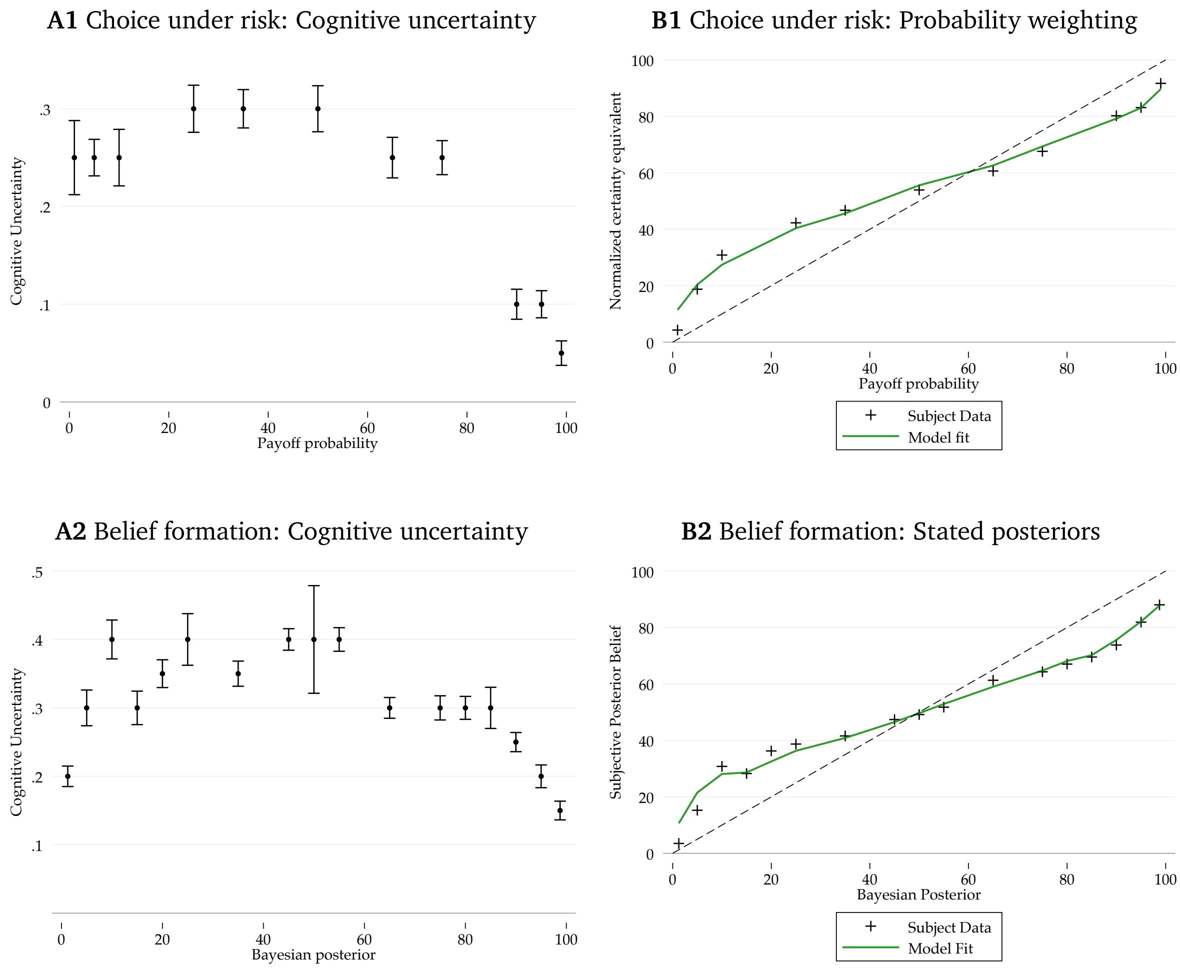


Figure 7: Left panels: median cognitive uncertainty as a function of probabilities. Right panels: fitted values of estimates of eq. (4), where for each decision problem a subject is assigned the across-subject average CU for this problem. Figure B2 displays bins with 30 or more observations. (Risk: $N = 4,524$, Beliefs: $N = 4,590$)

choice under risk, we compute the average level of cognitive uncertainty for each payout probability, and then estimate the model based on these average levels of CU. This is justified for the purposes of the present exercise because here our focus is precisely on the variation in cognitive uncertainty *across objective probabilities* rather than across subjects.

The right-hand panels of Figure 7 show the fit of these model estimates. We see that the amended reduced-form model accounts for the non-linearity in decisions and attributes it partly to the hump-shaped relationship between cognitive uncertainty and objective probabilities.

E Details on B Experiments

We here briefly summarize the "B" experiments that formed the core of our earlier NBER working paper with the same title (see Table 1). For more details please refer to Enke and Graeber (2019).

E.1 Decision Tasks

Choice under risk. In experiment *Risk B*, we followed a large set of previous works and implemented multiple price lists that elicit certainty equivalents for lotteries (see, e.g. Tversky and Kahneman, 1992; Bruhin et al., 2010; Bernheim and Sprenger, 2019). Each subject completed a total of six price lists. On the left-hand side of the decision screen, a simple lottery was shown that paid \$$y$ with probability p and nothing otherwise. On the right-hand side, a safe payment \$$z$ was offered that increased by \$1 for each row that one proceeds down the list. As in Bruhin et al. (2010) and Bernheim and Sprenger (2019), the end points of the list were given by $z = 0$ and $z = y$.

Throughout, we did not allow for multiple switching points. This facilitates a simpler elicitation of cognitive uncertainty. To enforce unique switching points, we implemented an auto-completion mode: if a subject chose Option A in a given row, the computer implemented Option A also for all rows above this row. Likewise, if a subject chose Option B in a given row, the computer instantaneously ticked Option B in all lower rows. However, participants could always revise their decision and the auto-completion before moving on.

The parameters y and p were drawn uniformly randomly and independently from the sets $y \in \{15, 20, 25\}$ and $p \in \{5, 10, 25, 50, 75, 90, 95\}$. We implemented both gain and loss gambles, where the loss amounts are the mirror images of y. In the case of loss gambles, the lowest safe payment was given by $z = -\$y$ and the highest one by $z = \$0$. In loss choice lists, subjects received a monetary endowment of \$$y$ from which potential losses were deducted. Out of the six choice lists that each subject completed, three concerned loss gambles and three gain gambles. We presented either all loss gambles or all gain gambles first, in randomized order.

Finally, with probability 1/3, a choice list was presented in a compound lottery format, as described in the main text.

Belief updating. The procedures in *Beliefs B* were essentially the same as in *Beliefs A*, with slight changes in the experimental instructions used.

Survey expectations. As in the main experiments, we elicited expectations about the 12-months return of the S&P 500. In addition, we also measured inflation expectations:

> *[Explanation of inflation rates.] We randomly picked a year X between 1980 and 2018. Imagine that, at the beginning of year X, the set of products that is used to compute the inflation rate cost $100. What do you think is the probability that, at the end of that same year, the same set of products cost less than $y? (In other words, what do you think is the probability that the inflation rate in year X was lower than z%?)*

Finally, we also elicited respondents' beliefs about the national income distribution:

> *Assume that in 2018, we randomly picked a household in the United States. What do you think is the probability that this household earned less than USD y in 2018, before taxes and deductions?*

E.2 Measuring Cognitive Uncertainty

Choice under risk. After stating a switching interval in a price list, a participant was reminded of their valuation (switching interval) for the lottery on the previous price list screen. They were then asked to indicate how certain they are that to them the lottery is worth exactly the same as their previously indicated certainty equivalent. To answer this question, subjects used a slider to calibrate the statement "I am certain that the lottery is worth between a and b to me." If the participant moved the slider to the very right, a and b corresponded to the previously indicated switching interval. For each of the 20 possible ticks that the slider was moved to the left, a decreased and b increased by 25 cents, in real time. In gain lotteries, a was bounded from below by zero and b bounded from above by the lottery's upside. Analogously, for losses, a was bounded from below by the lottery's downside and b from above by zero. The slider was initialized at cognitive uncertainty of zero, but subjects had to click somewhere on the slider in order to be able to proceed.

Belief updating. The instructions introduced the concept of an "optimal guess." This guess, we explained to subjects, uses the laws of probability to compute a statistically correct statement of the probability that either bag was drawn, based on Bayes' rule. We highlighted that this optimal guess does not rely on information that the subject does not have.

After subjects had indicated their probabilistic belief that either bag was drawn, the next decision screen elicited cognitive uncertainty. Here, we asked subjects how certain they are that their own guess equals the optimal guess for this task. Operationally, similarly to the case of choice under risk, subjects navigated a slider to calibrate the statement "I am certain that the optimal guess is between a and b.", where a and b collapsed to the subject's own previously indicated guess in case the slider was moved to the very right. For each of the 30 possible ticks that the slider was moved to the left, a decreased and b increased by one percentage point. a was bounded from below by zero and b bounded from above by 100. Again, the slider was initialized at cognitive uncertainty of zero and we forced subjects to click somewhere on the slider to be able to proceed.

E.3 Logistics and Pre-Registration

Based on a pre-registration, we recruited $N = 700$ completes. We restricted our sample to AMT workers that were registered in the United States, but we did not impose additional participation constraints. After reading the instructions, participants completed three comprehension questions. Participants who answered one or more control questions incorrectly were immediately routed out of the experiment and do not count towards the number of completes. In addition, towards the end of the experiment, a screen contained a simple attention check. Subjects that answered this attention check incorrectly are excluded from the data analysis and replaced by a new complete, as specified in the pre-registration. In total, 62% of all prospective participants were screened out of the experiment in the comprehension checks. Of those subjects that passed, 2% were screened out in the attention check.

In terms of timeline, subjects first completed six of the choice under risk tasks. Then, we elicited their survey expectations about various economic variables, as discussed below. Finally, participants completed a short demographic questionnaire and an eight-item Raven matrices IQ test.

Participants received a completion fee of \$1.70. In addition, each participant earned a bonus. The experiment comprised three financially incentivized parts: the risky choice lists, the survey expectations questions, and the Raven IQ test. For each subject, one of these parts of the experiment was randomly selected for payment. If choice under risk was selected, a randomly selected decision from a randomly selected choice list was paid out.

The experiments reported in this appendix were pre-registered in the AEA RCT registry, see `https://www.socialscienceregistry.org/trials/4493`. As we pre-registered, all regression analyses of the replication data exclude extreme outliers. In choice under risk, these are observations for which (i) the normalized certainty equivalent is strictly larger than 95% while the objective payout probability is at most 10%, or (ii) the normalized certainty equivalent is strictly less than 5% while the objective payout probability is at least 90%. In belief updating, outliers are defined analogously.

E.4 Results for Choice Under Risk

Table 14 provides a regression analysis of the data. As in the main paper, our object of interest is the extent to which a subject's normalized certainty equivalent is (in)sensitive to variations in the probability of the non-zero payout state. Thus, we regress a participant's absolute normalized certainty equivalent on (i) the probability of receiving the non-zero gain / loss; (ii) cognitive uncertainty; and (iii) an interaction term.

The results show that higher cognitive uncertainty is associated with lower responsiveness to variations in objective probabilities, in both the gains and the loss domain. In terms of quantitative magnitude, the regression coefficients suggest that with cognitive uncertainty of zero, the slope of the neo-additive weighting function is given by 0.65, yet it is only 0.34 for maximum cognitive uncertainty of one. A different way to gauge quantitative magnitudes is to standardize cognitive uncertainty into a z-score. When doing so, the regression results (not reported) suggest that an one standard deviation increase in cognitive uncertainty decreases the slope of the neo-additive weighting function by about 0.11. These are arguably large effect sizes that underscore the quantitative relevance of cognitive uncertainty in generating probability weighting.

Table 14: Inelasticity with respect to probability and cognitive uncertainty in *Risk B*

	Dependent variable: Absolute normalized certainty equivalent					
	Gains		Losses		Pooled	
	(1)	(2)	(3)	(4)	(5)	(6)
Probability of payout	0.68***	0.68***	0.59***	0.59***	0.65***	0.65***
	(0.02)	(0.02)	(0.03)	(0.03)	(0.02)	(0.02)
Probability of payout × Cognitive uncertainty	-0.41***	-0.41***	-0.20**	-0.20**	-0.31***	-0.31***
	(0.09)	(0.09)	(0.09)	(0.09)	(0.07)	(0.07)
Cognitive uncertainty	11.6**	11.3**	14.8***	14.8***	13.5***	13.5***
	(5.19)	(5.18)	(5.26)	(5.17)	(3.84)	(3.84)
Demographic controls	No	Yes	No	Yes	No	Yes
Observations	1271	1271	1254	1254	2525	2525
R^2	0.54	0.54	0.41	0.41	0.47	0.47

Notes. Demographic controls include age, gender, college education and performance on the Raven matrices test. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. The dependent variable is a subject's absolute normalized certainty equivalent. The sample includes choices from all baseline gambles with strictly interior payout probabilities. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

E.5 Results For Belief Updating

Columns (1)–(3) of Table 15 provide an econometric analysis, which again corresponds to the neo-additive weighting function. Here, we regress a subject's stated posterior on (i) the Bayesian posterior; (ii) cognitive uncertainty; and (iii) their interaction term. We find that with cognitive uncertainty of zero, the slope of the neo-additive weighting function is given by 0.83 but it is only 0.41 with cognitive uncertainty of one.

Table 15: Belief updating: Regression analyses for *Beliefs B*

	Dependent variable:					
	Posterior belief		Ln [Posterior odds]			
	(1)	(2)	(3)	(4)	(5)	(6)
Bayesian posterior	0.80***	0.80***				
	(0.01)	(0.01)				
Bayesian posterior × Cognitive uncertainty	-0.39***	-0.39***				
	(0.04)	(0.04)				
Cognitive uncertainty	16.6***	16.5***	-0.17**	-0.17**	-0.16**	-0.16**
	(2.32)	(2.32)	(0.07)	(0.07)	(0.07)	(0.07)
Log[Posterior Odds]			0.50***	0.50***	0.58***	0.58***
			(0.01)	(0.01)	(0.03)	(0.03)
Ln [Bayesian odds] × Cognitive uncertainty			-0.24***	-0.24***		
			(0.04)	(0.04)		
Log [Likelihood ratio]					-0.099**	-0.10**
					(0.04)	(0.04)
Log [Prior odds] × Cognitive uncertainty					-0.40***	-0.40***
					(0.07)	(0.07)
Log [Likelihood ratio] × Cognitive uncertainty					-0.20***	-0.19***
					(0.05)	(0.05)
Constant	11.1***	11.2***	0.046	0.036	0.043	0.035
	(0.90)	(1.86)	(0.03)	(0.10)	(0.03)	(0.10)
Demographic controls	No	Yes	No	Yes	No	Yes
Observations	3187	3187	3012	3012	3012	3012
R^2	0.73	0.73	0.63	0.63	0.63	0.63

Notes. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. To avoid a mechanical loss of observations resulting from the log odds definition, the log posterior odds in columns (3)–(6) are computed by replacing stated posterior beliefs of 100% and 0% by 99% and 1%, respectively. The results are virtually identical without this replacement. Demographic controls include age, gender, college education and performance on a Raven matrices test. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

E.6 Results for Economic Forecasts

Table 16 summarizes the results. Again, we see that the responsiveness of stated expectations with respect to the objective / historical probabilities strongly decreases in measured cognitive uncertainty.

Table 16: Survey expectations: Regression analyses for the B experiments

	Dependent variable: Probability estimate about:					
	Income distr.		Stock market		Inflation rate	
	(1)	(2)	(3)	(4)	(5)	(6)
Objective probability	0.90***	0.90***	0.69***	0.69***	0.76***	0.76***
	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)	(0.02)
Objective probability × Cognitive uncertainty	-0.41***	-0.41***	-0.53***	-0.52***	-0.60***	-0.60***
	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)
Cognitive uncertainty	18.9***	18.9***	24.2***	24.6***	27.5***	27.4***
	(2.37)	(2.37)	(2.27)	(2.31)	(2.86)	(2.86)
Demographic controls	No	Yes	No	Yes	No	Yes
Observations	1980	1980	1892	1892	1848	1848
R^2	0.83	0.83	0.52	0.52	0.54	0.54

Notes. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. Demographic controls include age, gender, college education and performance on the Raven matrices test. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

F Partition Manipulation

Under the assumptions that (i) the model parameter d reflects a fixed prior and (ii) that it is partly influenced by a 1/N logic, where N is the number of states, we can manipulate the default / prior through a so-called partition manipulation that increases the number of states without changing the normatively relevant problem features. If this actually affects the prior, the model in Section 2 would predict that observed decisions decrease, and that the magnitude of this effect increases in cognitive uncertainty.

Design. We designed treatment conditions that increase the number of states from two to ten. We further designed these treatments with the objective of holding cognitive uncertainty fixed. In choice under risk, we do so by framing probabilities in terms of the number of colored balls in a bag. For example, we describe a lottery as:

> Out of 100 balls, 80 are red. If a red ball gets drawn: get \$20.
> 20 balls are blue. If a blue ball gets drawn: get \$0.

In addition to this treatment, labeled *Risk A high default*, we also implemented treatment *Risk A low default* in a within-subjects design (with random order of treatments). Here, we implemented the same lotteries as in *Risk A high default*, yet we split the zero-payout state into nine payoff-equivalent states with different probability colors. For example, the lottery above would be described as:

> Out of 100 balls, 80 are red. If a red ball gets drawn: get \$20.
> 2 balls are blue. If a blue ball gets drawn: get \$0.
> 2 balls are black. If a black ball gets drawn: get \$0.
> 2 balls are white. If a white ball gets drawn: get \$0.
> ...
> 4 balls are yellow. If a yellow ball gets drawn: get \$0.

We designed a similar manipulation for the balls-and-urns updating task. Recall that in treatment *Beliefs A*, an example updating problem is that the base rates for bags A and B are 70% and 30%, and the signal diagnosticity (number of red balls in bag A and number of blue balls in bag B) is 70%. Now, in treatment *Beliefs A low default*, we split the probability mass for bag B up into nine different bags. That is, there are now ten bags, labeled A through J. In the specific example above, the base rate for A would again be 70%, the one for B through I 3% each and the one for J 6%. Bag A would contain 70 red and 30 balls, and all bags B through J 30 red and 70 blue balls. That is, these bags have identical ball compositions. Observe that in both the lottery choice problem and the belief updating task the normatively relevant structure of the problem (which consists of the objective lottery payoff profile and the Bayesian posterior) is held constant.

Importantly, the elicitation of decisions was held exactly constant across treatments. For instance, in both *Beliefs A replication* and *Beliefs A low default*, subjects only enter their subjective probability that Bag A got selected. The implied probability for the other events was displayed automatically. In *Beliefs A replication*, if a subject entered probability p% for Bag A, then our computer interface automatically showed the joint subjective probability for Bags B–J as $(1-p)$%.

We implemented these experiments in two ways. First, as part of the A experiments, with a total of 102 subjects in lottery choice and 108 subjects in belief updating. This was conducted in a within-subjects design. Second, we implemented the same treatments as part of our B experiments. The only differences to the A experiments were (i) as always, a different CU measure, (ii) a multiple price list procedure rather than a direct elicitation, and (iii) that in experiment B the treatments were implemented in a between-subjects design. 300 subjects participated in *Risk B low default* and *Risk B high default*. 300 subjects participated in treatment *Beliefs B low default*, which was randomized within the same experimental sessions as a replication of treatment *Beliefs B*. Appendix G shows screenshots of the experimental instructions. Appendix E.3 discusses the pre-registration of the B experiments, including the pre-specified exclusion of extreme outliers.

Interpretation. This experimental manipulation lends itself to two interpretations, both of which we embrace. First, as discussed above, the default decision could be influenced by a type of ignorance prior or 1/N heuristic. A second interpretation is that the manipulation makes the zero-payout state in the risky choice problems more visually salient because it now appears nine times on the decision screen. Similarly, in the belief updating problems, Bag A (the one for which we elicit the subjective posterior probability) becomes less salient because there are nine other bags in the partition manipulation. These two interpretations share the common theme that they emphasize how the partition manipulation changes people's heuristic (or intuitive) response, prior to actually thinking about the specific problem at hand. This is what we intend to capture and manipulate.

Results. We report mixed results across the A and B experiments. First, we find that increasing the number of partitions significantly increases cognitive uncertainty in the belief updating A experiment (two-sided t-test, $p = 0.01$), but not in the risk A ($p = 0.73$), choice under risk B ($p = 0.98$) or beliefs B experiments ($p = 0.39$). In all variants, average cognitive uncertainty in the high-partition treatment exceeds average cognitive uncertainty in the low-partition treatments. This suggests that the experimental manipulation affects cognitive noise to some extent.

Second, Appendix Table 17 reports regression estimates for the effect on choices. We find that the partition manipulation significantly decreases average decisions in all experiments but the *Risk A* variant. Moreover, in the B experiments on belief updating and choice under risk, in both of which the manipulation affected average choices, the response to the change in the default is significantly more pronounced among cognitively uncertain decisions. In the A experiments, the interaction between the treatment manipulation and cognitive uncertainty is very weak, though note that in *Risk A* we do not observe a main treatment effect on choices to begin with. Overall, we conclude that the partition manipulation is only partly successful in affecting the cognitive default without affecting cognitive noise.

Table 17: Partition manipulations

Dependent variable:	*Risk A* Normalized CE			*Beliefs A* Posterior belief			*Risk B* Normalized CE			*Beliefs B* Posterior belief		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
0 if *Baseline*, 1 if *Low Default*	-1.04	-0.95	-1.00	-10.4***	-8.58**	-8.57**	-11.2***	-8.05***	-7.64***	-4.30***	-0.62	-0.65
	(1.42)	(1.99)	(2.04)	(2.30)	(3.70)	(3.68)	(2.06)	(2.61)	(2.61)	(1.06)	(1.82)	(1.82)
1 if *Low Default* × Cognitive uncertainty		-0.049	0.20		-2.93	-2.97		-15.6**	-17.0**		-11.7***	-11.7***
		(6.47)	(6.58)		(9.93)	(9.82)		(7.84)	(7.96)		(3.61)	(3.62)
Cognitive uncertainty		-18.6***	-20.4***		-26.6***	-26.5***		-0.70	0.22		-3.42**	-3.45*
		(6.75)	(6.89)		(7.21)	(7.07)		(6.64)	(6.78)		(1.72)	(1.77)
Demographic controls	No	No	Yes	No	No	Yes	No	No	Yes	No	No	Yes
Observations	1224	1224	1224	1296	1296	1296	881	881	881	5372	5372	5372
R^2	0.00	0.02	0.03	0.02	0.05	0.05	0.04	0.06	0.07	0.00	0.01	0.01

Notes. Demographic controls include age, gender, college education and performance on the Raven matrices test. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. All regressions exclude extreme outliers as pre-registered and discussed in Appendix Section E.3. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

G Experimental Instructions and Control Questions

Below we provide screenshots of the instructions, control questions and decision screens of the main experiments. Corresponding information for the self-replication experiments as well as the default manipulation experiments can be found in a working paper version of this paper (Enke and Graeber, 2019).

G.1 Treatment *Risk Main*

Part 1: Instructions (1/3)

Please read these instructions carefully. "If you correctly answer three qualification questions and complete this study you will additionally receive a fixed bonus of $1.50.

In this study, there are various lotteries with different probabilities of winning. An example is:

With probability **5%: Get $ 18**
With probability **95%: Get $ 0**

The lotteries will actually be played out by the computer and determine your earnings in exactly the way we describe on the following pages.

Decision screen 1

You will be asked to choose which of two payment options you prefer: a lottery or a payment that you would receive *with certainty*. For the lottery displayed above, for example, you would be asked the following list of questions:

<table>
<tr><th>Question #</th><th></th><th>Option A</th><th></th><th>Option B</th></tr>
<tr><td>1</td><td>Would you rather have:</td><td rowspan="6">With probability 5 % : Get $18
With probability 95 %: Get $0</td><td>or</td><td>$0.01 with certainty</td></tr>
<tr><td>2</td><td>Would you rather have:</td><td>or</td><td>$0.02 with certainty</td></tr>
<tr><td>3</td><td>Would you rather have:</td><td>or</td><td>$0.03 with certainty</td></tr>
<tr><td>...</td><td>...</td><td>...</td><td>...</td></tr>
<tr><td>1,799</td><td>Would you rather have:</td><td>or</td><td>$17.99 with certainty</td></tr>
<tr><td>1,800</td><td>Would you rather have:</td><td>or</td><td>$18.00 with certainty</td></tr>
</table>

In each question you pick either Option A (the lottery) or Option B (the certain payment). We will randomly pick one question and pay you according to what you chose on that one question. Each question is equally likely to be chosen for payment.

We assume that you prefer the lottery (Option A) when the certain payment in Option B is very small, but that you switch to preferring Option B when the certain payment is very large. Therefore, to save time, we will not actually ask you 1,800 questions. Instead, we will simply ask you to state the amount at which you would switch from Option A to B in the table. This is the same as asking you:

Which certain payment is worth as much to you as this lottery?

Based on your response to this question, we can then fill out your answers to all 1,800 questions in the list. That is, you will choose Option A for all questions with a lower certain payment than your response, and Option B for all questions with a certain payment that is at least as high as your response.

Example: Suppose you were to tell us that a given lottery is worth as much to you as a certain payment of $12. Then this would mean that you prefer the lottery over any certain payment that is smaller than $12. On the other hand, it would also mean that you prefer any certain payment greater than $12 over the lottery.

Next

Part 1: Instructions (2/3)

Example

With probability **35% : Get $ 18**
With probability **65% : Get $0**

Which certain payment is worth as much to you as this lottery?

Getting $ ____ with certainty is worth as much to me as this lottery.

Next

Part 1 of this study: Instructions (3/3)

Decision screen 2: Your certainty about your decision

When you make your decision, you may feel **uncertain about which certain payment is worth as much to you as the lottery**. On decision screen 2, we will ask you to select a button to indicate **how certain** you are that you value the lottery within +/- $0.50 of the amount you entered on the previous screen.

Example

Suppose that on the first decision screen you indicated that you value a 5% chance of getting $18 as much as receiving $5.50 with certainty. Your second decision screen would look like this:

With probability **5% : Get $18**
With probability **95% : Get $0**

How certain are you that you actually value this lottery somewhere between getting $5.00 and $6.00?

○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
0%	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%	100%

very uncertain **completely certain**

Next

Qualification Questions

Important: If you answer any of the below questions incorrectly, the study ends after this page and you will receive your participation payment, but you do not qualify for a bonus.

1. Which one of the following statements is correct if the following lottery is played for you?

With probability **60%: Get $ 15**
With probability **40%: Get $ 5**

Please select one of the statements:

- ○ It is possible that I get paid both $15 and $5, i.e., I may receive a total amount of $20 from this lottery.
- ○ I receive EITHER $15 OR $5 from this lottery.
- ○ It is possible that I receive no money from this lottery.

2. Jon is faced with the following lottery:

Decision Screen (1/2)

With probability **10% : Get $ 20**
With probability **90% : Get $ 0**

Which certain payment is worth as much to you as this lottery?

Getting $ [   ] with certainty is worth as much to me as this lottery.

Next

Suppose Jon enters $13. Which one of the following statements is correct? :

- ○ Jon would prefer $12 over the lottery.
- ○ Jon would prefer the lottery over any certain payment.
- ○ Jon would prefer the lottery over $10.

3. Now imagine that Jon is uncertain about how much exactly the lottery is worth to him.
Jon; however, is 60% certain that the lottery is worth between $12.5 and $13.5 to him. Please select the correct button to accurately reflect this level of certainty:

○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
0%	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%	100%

very uncertain **completely certain**

Next

Task 1 of 12

Decision Screen (1/2)

With probability **65% : Get $ 22**
With probability **35% : Get $ 0**

Which certain payment is worth as much to you as this lottery?

Getting $ ______ with certainty is worth as much to me as this lottery.

Next

Task 1 of 12

Decision Screen (2/2)

With probability **65% : Get $ 22**
With probability **35% : Get $ 0**

Your decision on the previous screen indicates that you value this lottery as much as getting $1.00 with certainty.

How certain are you that you actually value this lottery somewhere between getting $0.50 and $1.50?

0%	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%	100%

very uncertain **completely certain**

Next

G.2 Treatment *Beliefs Main*

Part 1 of this Study: Instructions (1/4)

Please read these instructions carefully. There will be bonus qualification questions based on the instructions. If you do not answer all bonus qualification questions correctly, you will not earn an additional bonus for this study.

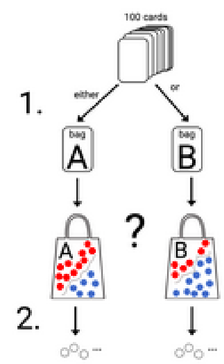
In this study, you will be asked to complete **12 guessing tasks**.
In each guessing task, there are two bags, "Bag A" and "Bag B". A bag contains exactly 100 balls, each of which is either red or blue. However, the two bags contain **different numbers of red vs. blue balls:** one bag contains more red balls, the other contains more blue balls. One of the bags is selected at random by the computer as described below. You will not observe which bag was selected. Instead, the computer will randomly draw one or several balls from the secretly selected bag, and will show these balls to you. Your task is to **guess which bag was selected** based on the available information. The exact procedure is described below:

Task setup

- There is a deck of cards that consists of 100 cards. Each card in the deck either has "Bag A" or "Bag B" written on it. You will be informed about **how many** of these 100 cards have "Bag A" or "Bag B" written on them.
- There are two bags, "Bag A" and "Bag B". Each bag contains 100 balls. One of the bags contains more red balls, and the other bag contains more blue balls. You will be informed about exactly **how many red and blue balls** each bag contains.

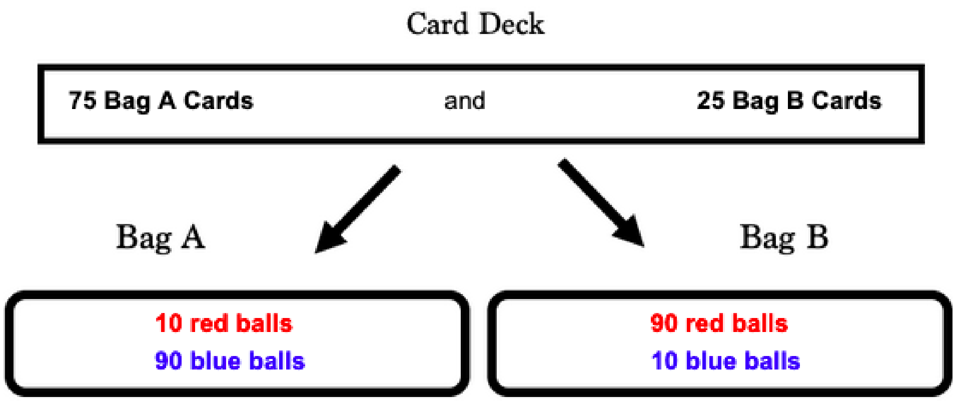
Sequence of events

1. The computer **randomly selects one** of the 100 cards, with equal probability.
   If a "Bag A" card was drawn, Bag A is selected.
   If a "Bag B" card was drawn, Bag B is selected.
2. Next, the computer **randomly draws one or more** balls from the secretly selected bag. Each ball is equally likely to get selected. Importantly, if more than one ball is drawn, the computer draws these balls **with replacement.** This means that after a ball has been drawn and taken out of the bag, it gets replaced by a ball of the same color. Thus, the probability of a red or blue ball being drawn does not depend on whether previous draws were red or blue.
   The computer will **inform you about the color** of the randomly drawn balls.

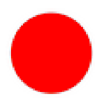


Instructions (2/4)

Here's an example of what a decision screen will look like. At the top of the screen, you see the information you need to solve the guessing task. At the bottom, you enter your guess.



The computer randomly selected a card from the deck and then randomly drew the following ball(s) from the selected bag:



Decision 1: Your guess

Given that these balls were drawn, **how likely** do you think it is that Bag A (as opposed to Bag B) has been selected (in %)?

I believe it is ____ % likely that Bag A was selected.

Submit your guess

Instructions (3/4)

The optimal guess

Using the laws of probability, the computer computes a **statistically correct statement of the probability that Bag A was selected**, based on all the information available to you. This **optimal guess** does not rely on information that you do not have. It is just the best possible (this means: payoff-maximizing) estimate given the available information. In technical terms, this guess is based on a statistical rule called Bayes' Law.

Decision screen 2: Your certainty about your guess

In any given task, you may actually be **uncertain about whether your probability guess corresponds to the optimal guess**. On decision screen 2, we will ask you to select a button to indicate **how certain** you are that the optimal guess is within ± 1 percentage point of your answer.

Example

Suppose that you stated that Bag A was selected with probability 80%. Your second decision screen would then look like this:

Your decision on the previous screen indicates that you believe there is a 80% chance of Bag A having been selected.

How certain are you that the optimal guess is somewhere between 79.0% and 81.0%?

0% 5% 10% 15% 20% 25% 30% 35% 40% 45% 50% 55% 60% 65% 70% 75% 80% 85% 90% 95% 100%

very uncertain **completely certain**

Next

Instructions (4/4)

Summary: Sequence of events in each task

You will be asked to complete 12 guessing tasks. For each task, there will be **2 decision screens:**

Decision screen 1

You will be asked to enter probabilities that express **how likely** you think it is that Bag A or Bag B has been selected.

The following aspects will vary across the different guessing tasks:

- How many Bag A cards and Bag B cards are contained in the card deck.
- How many red and blue balls are contained in bags A and B.
- The number and color of the drawn balls.

Decision screen 2

You will be asked to indicate **how certain** you are that the guess you provided on decision screen 1 **is close to the optimal guess in this task**.

Your payment for part 1

You can potentially earn $5.00 with your guess. Your probability of winning $5.00 is higher the higher the probability you assign to the bag that actually got selected. If Part 1 determines your payment, the computer will randomly select one of your guesses to be relevant for your payment.

To maximize your earnings, you should therefore simply try to guess as accurately as possible which bag was selected. In case you're interested, the specific formula that determines whether you get the prize is explained here.

Next

Qualification Questions

Important: If you answer any of the below questions incorrectly, the study ends after this page and you will receive your participation payment, but you do not qualify for a bonus.

1. Which statement about the number of cards corresponding to each bag is correct?
   - ◯ The number of "Bag A" cards is the same in all tasks.
   - ◯ The exact number of cards corresponding to each bag may vary across tasks.
2. Suppose there are 70 Bag A cards and 30 Bag B cards in the deck. Further suppose there are more red balls in Bag A than there are red balls in Bag B. Next, one red ball is drawn from the secretly selected bag.
   Which one of the following guesses is closest to the statistically optimal guess?
   - ◯ 25%
   - ◯ 50%
   - ◯ 80%
3. Suppose that in a given task you guess that the probability that Bag A was selected is 70%. Further suppose you are 60% certain that the optimal guess is actually somewhere between 69% and 71% (close to your guess). Please select the appropriate button below to indicate this level of certainty.

◯	◯	◯	◯	◯	◯	◯	◯	◯	◯	◯	◯	◯	◯	◯	◯	◯	◯	◯	◯	◯
0%	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%	100%

very uncertain **completely certain**

Next

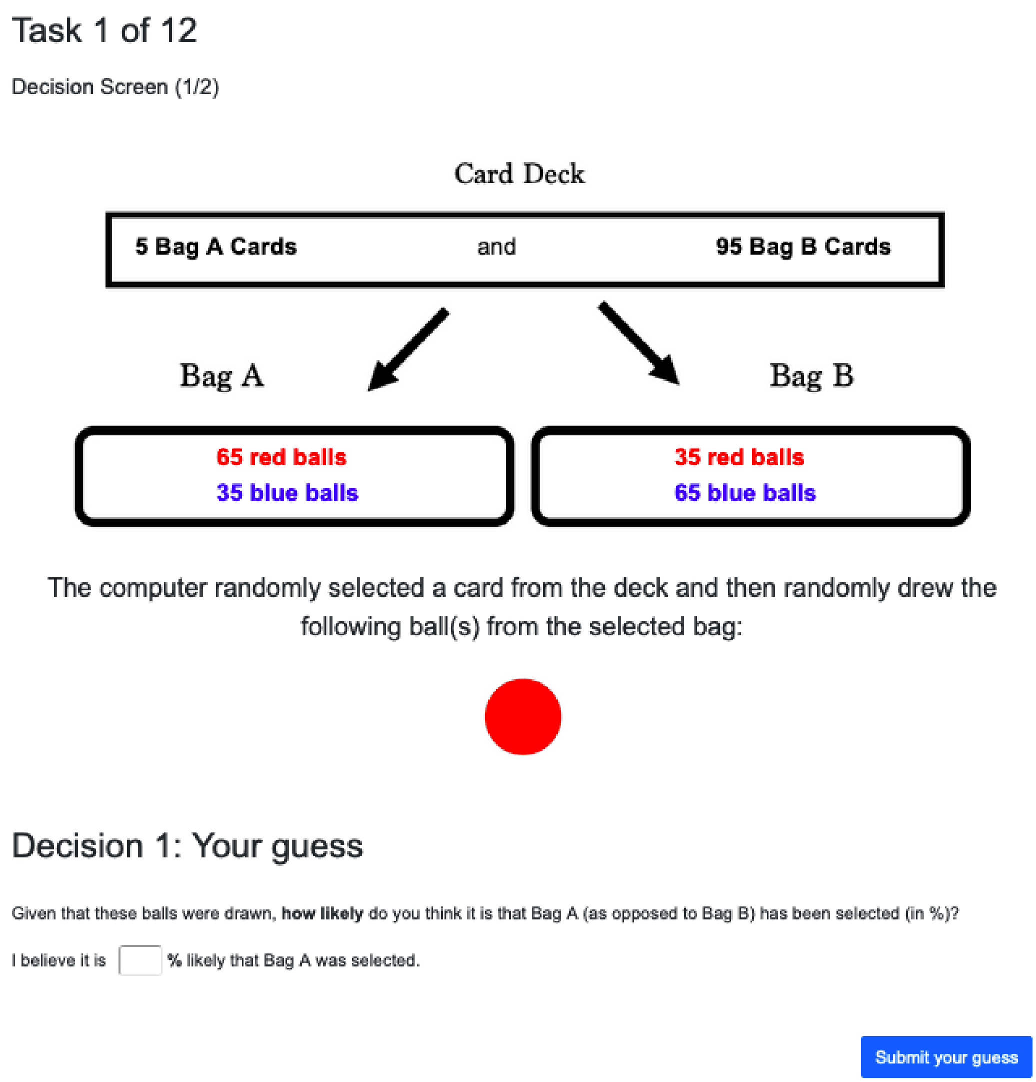


Figure 8: Beliefs elicitation

Task 1 of 12

Decision Screen (2/2)

Card Deck

5 Bag A Cards and **95 Bag B Cards**

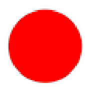
Bag A

65 red balls
35 blue balls

Bag B

35 red balls
65 blue balls

The computer randomly selected a card from the deck and then randomly drew the following ball(s) from the selected bag:



Decision 2: Your certainty

Your decision on the previous screen indicates that you believe there is a 1.0% chance that Bag A was selected.

How certain are you that the optimal guess is somewhere between 0.0% and 2.0%?

0% 5% 10% 15% 20% 25% 30% 35% 40% 45% 50% 55% 60% 65% 70% 75% 80% 85% 90% 95% 100%

very uncertain **completely certain**

Next

G.3 Stock market expectations

Question about the Stock Market

The S&P 500 is an American stock market index that includes 500 of the largest companies based in the United States.

Jon invested $100 in the S&P 500 today.
What is the **percent chance** that the **value of his investment will be less than $ 123** in one year from now?

In other words, what do you think is the percent chance that Jon will gain less than $23.0 or lose money on his investment over the next year?

Percent chance that Jons investment will be worth **less than $ 123** : %

Next

Your certainty about your estimate

On the previous screen, you indicated that you think there's a 1.0 % chance that a $100 investment into the S&P 500 today will be worth less than $123 in one year from now.

How certain are you that the statistically optimal guess (given the information you have) is somewhere between 0.0% and 2.0%?

0% 5% 10% 15% 20% 25% 30% 35% 40% 45% 50% 55% 60% 65% 70% 75% 80% 85% 90% 95% 100%

very uncertain **completely certain**

Next

G.4 Complex numbers in choice under risk

Information

In the next few rounds, there will be an additional complication. **In the following lotteries some of the amounts and/or probabilities may be expressed as mathematical expressions.** The lotteries will still be played out by the computer in exactly the way we describe.

Example

With probability **(6 x 9) / 3 - 7% : Get $ 18**
With probability **100 - ((6 x 9)/ 3 - 7)% : Get $0**

Which certain payment is worth as much to you as this lottery?

Getting $[ ] with certainty is worth as much to me as this lottery.

Task 7 of 12

Decision Screen (1/2)

With probability **(6 x 14) / 6 + 61% : Get $ 22**
With probability **100 - ((6 x 14) / 6 + 61)% : Get $ 0**

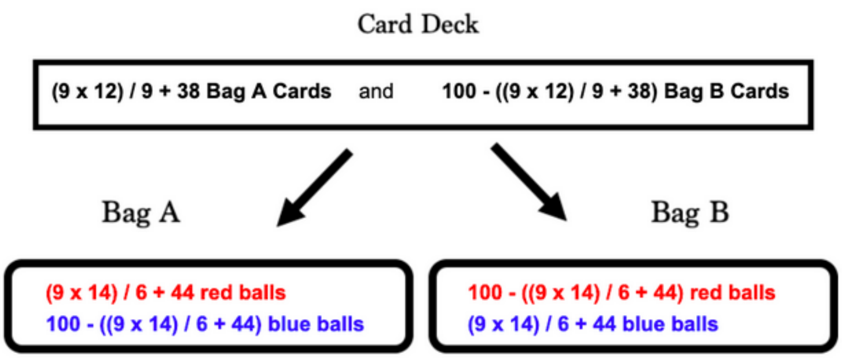
Which certain payment is worth as much to you as this lottery?

Getting $[ ] with certainty is worth as much to me as this lottery.

G.5 Complex numbers in belief updating

Information:

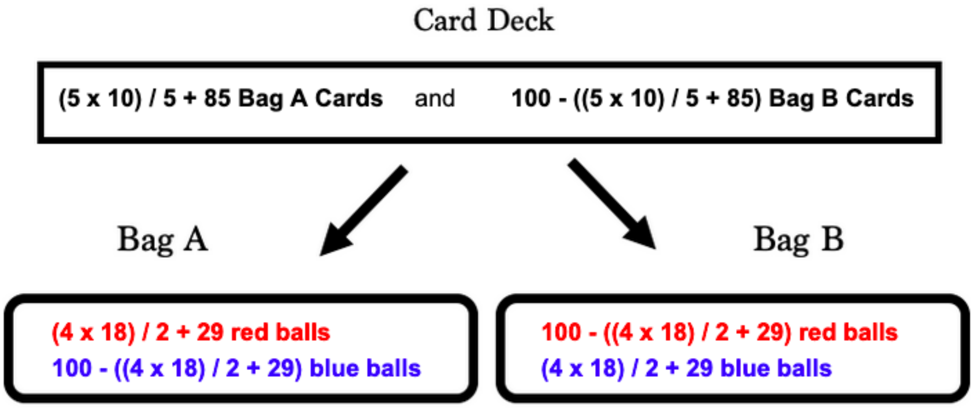
In the following rounds the information (i) on the number of Bag A and Bag B cards in the deck and (ii) on the number of red and blue balls in bags A and B will be expressed as mathematical expressions. The scenarios will still be played out by the computer in exactly the way we described. An example is:



The computer randomly selected a card from the deck and then randomly drew the following ball(s) from the selected bag:

Task 7 of 12

Decision Screen (1/2)



The computer randomly selected a card from the deck and then randomly drew the following ball(s) from the selected bag:

Decision 1: Your guess

Given that these balls were drawn, **how likely** do you think it is that Bag A (as opposed to Bag B) has been selected (in %)?

I believe it is [ ] % likely that Bag A was selected.

G.6 Default manipulation in choice under risk

Task 1 of 12

Decision Screen (1/2)

90 out of 100 balls are **red**. If one is drawn: Get **$20**.

10 out of 100 balls are **blue**. If one is drawn: Get **$0**.

Which certain payment is worth as much to you as this lottery?

Getting $ [   ] with certainty is worth as much to me as this lottery.

Next

Task 7 of 12

Decision Screen (1/2)

50 out of 100 balls are **red**. If one is drawn: Get **$25**.

5 out of 100 balls are **blue**. If one is drawn: Get **$0**.

5 out of 100 balls are **green**. If one is drawn: Get **$0**.

5 out of 100 balls are **orange**. If one is drawn: Get **$0**.

5 out of 100 balls are **brown**. If one is drawn: Get **$0**.

5 out of 100 balls are **pink**. If one is drawn: Get **$0**.

5 out of 100 balls are **black**. If one is drawn: Get **$0**.

5 out of 100 balls are **gold**. If one is drawn: Get **$0**.

5 out of 100 balls are **gray**. If one is drawn: Get **$0**.

10 out of 100 balls are **purple**. If one is drawn: Get **$0**.

Which certain payment is worth as much to you as this lottery?

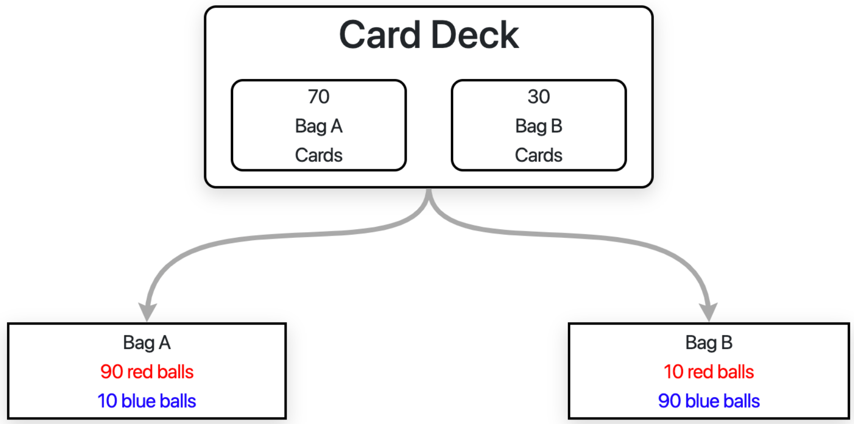
Getting $ [   ] with certainty is worth as much to me as this lottery.

Next

G.7 Default manipulation in belief updating

Task 1 of 12

Decision Screen (1/2)



The computer randomly selected a card from the deck and then randomly drew the following balls from the selected bag:

Decision 1: Your guess

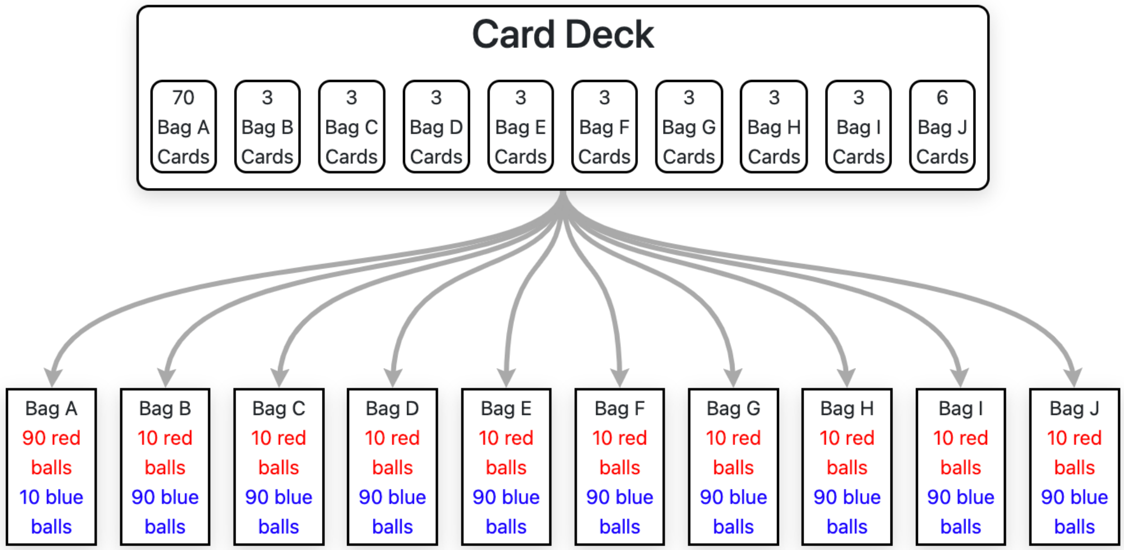
Given that these balls were drawn, **how likely** do you think it is that Bag A (as opposed to Bag B) has been selected (in %)?

I believe it is ☐ % likely that Bag A was selected.

Submit your guess

Task 7 of 12

Decision Screen (1/2)



The computer randomly selected a card from the deck and then randomly drew the following balls from the selected bag:

Decision 1: Your guess

Given that these balls were drawn, **how likely** do you think it is that Bag A (as opposed any of the Bags B, C etc. up to J.) has been selected (in %)?

I believe it is [ ] % likely that Bag A was selected.

Submit your guess